# Estimating Credit Rating Transition Probabilities for Corporate Bonds

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ABSTRACT

In this paper we take the task of motivating and exhibiting the potential of conditioning on economywide state variables in improving the forecasting of the Credit Rating Transition Probability (CRTP) Matrix. The improvement in CRTP matrix forecasting accuracy by utilizing state variable information is significant, both statistically and economically, in in-sample and out-of-sample experiments.

As a byproduct of examining the one-step ahead forecasting power of state variables in the information set of the researcher we undertake a study of the variation of credit migration (which includes default risk) over the business cycle. We find that an increase in nominal short and long and real rates, a lower equity return and a higher equity return volatility are associated with higher relative downgrade intensities. We compare our results with the predictions of credit risk models.

We deal with the implications of possible non-Markovian behavior of the credit rating process. Issuer-specific information is introduced in the form of time dependence and rating momentum. We examine whether the information provided by the state variables is subsumed or is augmented by incorporating various parametric or nonparametric types of time and occurrence dependence in the credit rating transition process. Even though there is significant evidence of time acceleration and downward rating momentum the pattern of state variable sensitivities remains mostly intact.

Taking into consideration industry classification reveals the different sensitivity of certain sectors to term structure variables with forecasting, relative value, possibly hedging and correlation implications. The financial sector in particular is more sensitive to interest rates.

Finally we examine unobservable to the econometrician heterogeneity which turns out not to alter the conclusions in an appreciable fashion.

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# I. Introduction

We can discern a clear tendency towards ‘derivatizing’ wider classes of risk. Among them credit risk is prominent, with the focus being on balance sheet management and diversification. Models to handle credit risk can be categorized according to the main purpose they are built. We can distinguish between models designed to price bank loans and corporate bonds, generally defaultable instruments (see, e.g. Duffie and Singleton, 1999), models attempting to capture the risk in portfolios of credit sensitive securities, where correlations and diversification effects have to be taken into account and finally models that are helping us to price credit derivatives.

A set of tools that has been widely developed in the last several years, applied for various risk management purposes and officially endorsed by the Basle Committee on Banking Supervision is Value-At-Risk analysis. It employs stress testing to calculate the exposure of the balance sheet of a financial institution to various events of an extreme nature.

The common thread between the VAR approach and the field of credit derivatives consists of their requirement of an accurate estimate of the transition probability matrix of agency (Moody’s and S&P, mainly) credit ratings. Models of the term structure of credit spreads based on a discrete state Markov chain of the default process, (e.g. Jarrow, Lando and Turnbull, 1997), are relying on the risk-neutral transformation of the empirical transition intensity matrix[2]. In addition, forecasting the rating evolution of CDO collateral is of paramount importance to extract the ‘diversity score’ of the underlying pool to be used for risk analysis and rating assignment. As far as extreme value testing is concerned, ‘migration analysis’ is employed to calculate the change of value in portfolios of bonds under alternative scenarios, where this standard deviation leads to the estimation of VAR due to credit risk exposure.

In this paper we take the task of motivating and exhibiting the potential of conditioning on economywide state variables in improving the forecasting of the Credit Rating Transition Probability (CRTP) Matrix. The improvement in CRTP matrix forecasting accuracy by utilizing state variable information is significant, both statistically and economically, in in-sample and out-of-sample experiments.

[2] The intensity-based default modeling approach as opposed to the original Black and Scholes (1974) formulation and their possible reconciliation are discussed, among others, in Duffie and Lando (1999).

As a byproduct of examining the one-step ahead forecasting power of state variables in the information set of the researcher we undertake a study of the variation of credit migration (which includes default risk) over the business cycle. We mainly examine term structure and equity return factors. We find that an increase in nominal short and long and real rates, a lower equity return and a higher equity return volatility are associated with higher relative downgrade intensities. We compare our results with the predictions of credit risk models.

Our next contribution consists of dealing with the implications of non-Markovian behavior of the credit rating process. Issuer-specific information is introduced in the form of time dependence and rating momentum. We examine whether the information provided by the state variables is subsumed or is augmented by incorporating various parametric or nonparametric types of time and occurrence dependence in the credit rating transition process. The overall pattern of correlations remains mostly intact. Examining the effect of taking into consideration industry classification reveals the different sensitivity of certain sectors to term structure variables with forecasting, relative value, possibly hedging and correlation implications.

In addition we examine the importance of unobservable heterogeneity, i.e. the importance of rating agency information that is not available to the econometrician and how it may alter the conclusions about the direction of state variable associations. On the one hand, we integrate the heterogeneity distribution out of the likelihood function. On the other hand we examine the impact of reincorporating heterogeneity information that we discarded in the interest of robustness in accordance with the literature. We conclude that unobservable to the econometrician heterogeneity does not alter the conclusions in an appreciable fashion.

We describe the approaches in the practitioner oriented and the admittedly sparse academic literature, in section II we outline the baseline constant intensity case, in section III we introduce state dependence, via economywide state variables, the estimation procedure is described and the forecasting problem is formulated, while we examine the variation of credit migration risk and the in and out-of-sample forecasting exercise in section IV. In section V we proceed by disentangling state from time dependence and detect possible departures from widely assumed non-Markovian behavior and in section VI we deal with observed and unobserved heterogeneity. Section VII concludes. The appendices include the description of the data and tables.

## 1. Literature Review

The rating agencies have been the exclusive providers of rating transition information for many years (see, e.g. Moody's, 1997 or Standard & Poor's, 1999). Further, the KMV Corporation has been a provider of individual firms distance to default measures, linking them to Expected Default Frequencies. In addition, Lucas and Lonski (1992) use Moody's dataset. Nickell et al (2000) and Kim (1999) present results and methodology relating to CRTPs conditioning. The estimation procedure followed by Kim (1999) and Nickell at al (2000) employs an ordered probit approach. It is well known that proceeding with the direct estimation of probabilities when the data for estimating transition intensities is available leads to inefficiency. In addition, Flinn and Heckman (1982) argue discrete time models' parameters and interpretation are dependent on the time interval used to perform estimation. At the same time, the parameters of continuous time models are invariant to the sampling time unit used to record observations. Moreover, conducting a yearly or quarterly probit estimation falls into the trap of neglecting duration and within year transition information. Further, the environment in the rating assignment procedure falls naturally in a discrete-state, continuous-time framework. On the one hand, the underlying creditworthiness of a debt issuing entity can be at any time fully described by its credit rating. On the other hand, everyday meetings of the rating committee ensure continual alertness.

Kim (1999) estimates conditional CRTP matrices quarter-by-quarter. In addition, he presents some evidence regarding his relative success in forecasting the CRTP matrix by conditioning on the speculative issuers' default rate and Nickell et al (2000) displays evidence on some significant difference between transition probabilities after accounting for differences in industry classification and domicile of the issuer. There is no study of credit migration risk and any out-of-sample exercise. Simulating a yearly 3-state Markov chain for GDP growth allows CRTP forecasting.

# II. Constant Intensity Estimation

## 1. Estimation methodology

The fundamental estimation object is the transition intensity from rating class i to rating class j is defined as the probability of departure to class j, in the next instant, given that you have survived in class i till time t and state variable $X_t$ is equal to X ($X_t$ belongs to the time t information set, it is predictable) [3].

$$\lambda_{ij}(X_t) = \lim_{dt \to 0} \frac{P(t \le T \le t + dt, J = j, X_t = X \mid T \ge t)}{dt} \tag{1}$$

We employ the proportional hazards assumption that parameterizes the intensities as

$$\lambda_{ij}(X_t) = k_1(X_t) k_2(t)$$

Initially we do not introduce state variables ($k_1(X_t) = k_1$) or time dependence ($k_2 = 1$) in the process driving credit rating transitions. We want to establish a benchmark for the estimation that will follow, which allows us to draw a comparison with the established rating agency practice. In essence we assume that the data is homogeneous, meaning that there are no regressors to make observations from a different, say, macroeconomic environment drawings from different distributions. Thus the purpose of this section is to illustrate the intensity construction and the likelihood calculations in this simplest case before introducing state variables and non-Markovian behavior. In addition, it serves to argue in favor of a constant intensity baseline in contrast to the rating agency approach.

We employ the 'competing risks' working hypothesis. Each study subject possesses an underlying failure time T, which is a random variable subject to censoring. When 'failure' occurs, which for our purposes encompasses downgrades, upgrades, and defaults and initially rating withdrawals, it can be of j-1 types (where j is the number of credit rating classes including Not Rated, NR[4]). Later, we treat rating withdrawals as censored observations[5].

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[3] For more details, see Lancaster 1990.

[4] According to the agencies' conventions, a rating of NR according to the S&P convention, or WR according to Moody's, reflects an interruption of credit quality monitoring due to non-credit related reasons, especially the maturing of an issue without any outstanding obligations remaining and mergers & acquisitions related events.

The probability of surviving is defined by cumulating the intensities of not failing for the instants that have elapsed during the lifetime of the study object defined as the negative of the hazard rate[6].

Hence we build a homogeneous Markov chain with infinitesimal generator A[7].

$$A = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & .... & \lambda_{1J} \\ \lambda_{21} & \lambda_{22} & \lambda_{231} & .... & \lambda_{2J} \\ .... & ... & .. & .... & ... \\ .... & ... & .. & .... & ... \\ \lambda_{J1} & \lambda_{J2} & \lambda_{J3} & .... & \lambda_{JJ} \end{bmatrix}, \lambda_{ii} = -\sum_{j=1, j\neq i}^{J} \lambda_{ij}$$

Our fundamental observation unit is the duration of stay in a particular rating. Maximizations are performed row by row taking into account ‘failures’ (upgrades or downgrades) in a competing risks framework from the particular rating that corresponds to the relevant row. We additionally accommodate censored observations. As durations are censored because of the discontinuation of the sample in December 31st 1998, we are dealing with simple Type II censoring, i.e. survival and censoring times are independent. In addition, rating withdrawals are interpreted as Type II censoring events. The likelihood function for the constant intensity case is formulated as follows. The contribution of duration l that involved a ‘failure’ to rating class j from rating class i at time t, when J-1 ‘failure’ types (competing risks) are possible is

$$\lambda_{ij} \exp(-\int_0^{t_l} \{ \sum_{k=1,k\neq i}^{J} \lambda_{ik} \} du) \tag{4}$$

---

[5] We can alternatively call those objects, cause-specific instantaneous hazard rates. Actually, the hazard rate is formally defined as the sum of the transition intensities over all possible failures, as the rate of ‘failing’ from a particular origin should reflect the sum of all the possible influences, i.e. $\lambda(t) = \sum_{j=1, j\neq i}^{J} \lambda_{ij}(t)$

[6] Accommodating Amemiya (1985), the probability of a study object staying in rating class I in period (0,t) (call this event A) can be obtained for small t, assuming Δt/t is an integer as $P(A) = \exp\{-(\sum_{j=1, j\neq i}^{J} \lambda_{ij})t\}$

[7] Jarrow et al (1997) treat default as an absorbing state, whereas in our estimation we can allow for non-zero entries in the last row of the generator. The data exhibits some cases of firms previously in default reentering capital markets. In addition, we check for positivity of eigenvalues to preclude the possibility of an aliasing problem.

This term constitutes the failure subdensity of our observation. In the censored case, failure has not happened yet so we dispose of the cause-specific hazard rate. Thus, with $N_j$ 'failures' to rating class j and $N_c$ censored observations, with original rating (i.e. row l) and N is equal to the sum of all observations, censored or uncensored,

$$L_l = \langle \prod_{j=1,\, j\neq l}^{J} \{\prod_{i=1}^{N_j} \lambda_j \exp(-\int_0^{t_i} \{ \sum_{j=1,\, j\neq l}^{J} \lambda_j \} du)\}\rangle \prod_{i=1}^{N_c} \exp(-\int_0^{t} \{ \sum_{j=1,\, j\neq l}^{J} \lambda_j \} du) \tag{5}$$

Taking first and second derivatives with respect to the log likelihood, which separates naturally we have

$$\lambda_j^{\ e} = \frac{N_j}{\sum_{i=1}^{N} t_i} \text{ ,which can be expressed as } \lambda_{ij}^{e} = \frac{\frac{N_{ij}}{N_i}}{\frac{\sum_{i=1}^{N_i} t_i}{N_i}} = \frac{\pi_{ij}}{\frac{\sum_{i=1}^{N_i} t_i}{N_i}} \text{ and } \operatorname{var}(\lambda_{ij}^{\ e}) = \frac{\lambda_{ij}^{\ e}}{\sum_{i=1}^{N} t_i}$$

The numerator is the maximum likelihood estimator of the probability that when departure occurs it is to j. The denominator is the mean time spent in the original rating class and is an estimator of the expected time spent in state i. The $\pi_{ij}$'s are the elements of the transition probability matrix of the discrete-state Markov chain imbedded in the continuous time Markov chain and are the appropriate quantities of interest if no duration data are available.

We derive the close connection between the constant intensity estimator for the intensity matrix and the nonparametric Kaplan-Meier estimator of the 1-month probability matrix P.

If we group the data into monthly intervals $I_1, \ldots, I_J$ and there are $m_j$ censored times and $d_{ikj}$ failure times from rating class i to rating class k. The estimators of the off-diagonal elements are the number of failures to the particular rating class over the potential ones, i.e. the ones that could have failed but did not.

$$p_{ikj} = \frac{d_{ikj}}{\sum_{k=1}^{\#ratings} \{d_{ikj} + m_{ij}\}}$$

We adjust for withdrawn ratings by employing the denominator correction proposed by Kalbfleisch and Prentice (1980)[8] and used by Moody's, with an appropriate scaling to have the rows summing to one[9]. The 'average' 1-month transition probability matrix is obtained by forming a weighted average of all monthly estimates. The weighting is done row-by-row taking into account the total number of firms that actually failed plus the ones that could have failed but did not. Thus,

$$p_{ij,t} = \frac{N_{ij,t}}{N_{j,t}}, \text{ which is weighted by } \frac{N_{j,t}}{\sum_{t=1}^{Months} N_{j,t}}$$

The denominator in the second expression is actually the sum of the survival times for the particular rating class. In this setting, the Kaplan-Meier estimator of the 1-month probability is equal to the estimator of the 1-month constant *intensity*. Moreover, the structure postulated lends itself naturally to a multinomial distribution characterization for the number of events during each time interval. In our framework, for each rating class in each interval there are $n_j$ independent experiments, which have k (number of rating classes) mutually exclusive termination ways, the probability of each outcome being $p_{ij}$. It can be shown that the standard error of any probability can be written as

$$Var(p_{ij}) = \frac{1}{n_{ij}}\{q_{ij}(1-q_{ij})\} \tag{7a}$$

---

8 Moody's estimator is in the spirit of the life table estimator of the conditional probability of failure in the relevant interval correction for censoring. Censoring is related to withdrawn ratings *during* the relevant time period. One can estimate the conditional probability of failure pj in the interval Ij, where mj is the number of censored observations, dj is the number of failures and nj is the number at risk at a time prior to tj, as

$$p_j = \frac{d_j}{n_j - \frac{m_j}{2}}$$

The correction in the denominator is an attempt to account for the fact that not all individuals are at risk for the whole interval.

9 Which is equivalent to subtracting the total number of censored observations in the denominator.

After estimating the intensities we calculate the transition probability matrix. The following version of the Kolmogorov equation holds for the constant intensity case where t will stand for the horizon.

$$\frac{dP(t)}{dt} = P(t)A \quad \text{, with the solution } P(t) = \exp(tA) \text{, approximated by } P(t) = \sum_{k=0}^{\infty} \frac{(tA)^k}{k!}$$

For our one-month baseline unit it turns that already the second order expansion does not improve the approximation meaningfully.

## 2. Results[10]

We present the one-year and the two-year transition probability matrices in the appendix with a left truncation correction[11] (Matrices 1a,c). Due to the exponential structure, they are easy to calculate. We compare the one-year version with the one provided by S&P (Matrix 1b). By using one-year static pools they are agnostic about the timing and intensity of rating activity within the year, effectively integrating out duration information by neglecting the occurrence and timing of within period transitions. This results in underestimating low-grade default probabilities and a slight overestimation of high-grade ones.

We also examine the question whether the estimated transition matrices satisfy monotone barrier likelihood requirements. As CreditMetrics© wonders, a common question one poses is, " what is the cumulative rate of crossing any given level of credit quality". We calculate the cumulative probabilities of any investment grade issue passing the speculative grade threshold (Table II.2.1). Rank order is preserved for all rating classes contrary to the CreditMetrics© calculation based on Moody's one-year transition matrix. The conditions analyzed in Kijima (1998) guarantee the satisfaction of this property for every barrier and horizon[12].

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[10] For tractability purposes we consolidate the ratings into 9 rating classes; the sign modifiers are lumped with the appropriate letter rating. See though the last section.

[11] See Appendix A.

[12] The intuition for the theorems therein reflects the fact that lower credit rating classes should be 'riskier' as they are expected to be absorbed in a smaller amount of time and that survival probabilities for lower rating classes are decreasing faster than the ones for higher rating classes.

## III. Stochastic Intensity Estimation

In this section we examine the effect of introducing state variables that modulate the intensities on CRTP forecasting relative to the constant intensity benchmark. In addition, we estimate how are the state variables associated with upgrade and downgrade intensities. Thus, we perform a credit migration risk study.

We distinguish first between factors relating with the state of the macroeconomy[13] in general and the credit markets in particular on the one hand and factors having to do with the composition of the existing issuer pool on the other hand. These are aggregate factors. Second, one needs to take firm-specific information into account. In this section we only deal with factors affecting all firms at the same time[14].

### a. Estimation Methodology

We have for the cause-specific instantaneous rate of failure[15] and the likelihood function[16]

$$\lambda_{ij}(X(t)) = \exp(\eta_{ij} + \gamma_i^u X(t))$$

$$L = \langle \prod_{j=1}^{J_u} \{ \prod_{i=1}^{N_j} \exp(\eta_j + \gamma^u X(t_i)) \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle \times$$

$$\langle \prod_{j=1}^{J_d} \{ \prod_{i=1}^{N_j} \exp(\eta_j + \gamma^d X(t_i)) \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle \times$$

$$\prod_{i=1}^{N_c} \exp(-\int_{\tau_i}^{t} \{ \sum_{j=1}^{J_u} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle$$

---

13 Credit markets feature a natural connection with the macroeconomic environment, but there appears to be a certain misalignment at some frequencies. As an example we point to the experience of the 1982 recession where the default rate was at a relatively low level. On the contrary, the fall of 1998 'liquidity crunch' manifested itself in various guises of credit market hardship, but was accompanied by benign general macroeconomic conditions.

14 We assume that the agencies account for firm-specific information in their rating class assignments, alternatively that the econometrician and the rating agency possess the same information set or that the rating agencies have more information but do not act upon it.

15 We restrict ourselves to the exponential form in order to ensure positivity.

16 $J_u$, ($J_d$, respectively) is the number of upgrade (downgrade) 'failures' and the integration lower limit involves the date that the duration was initiated, $\tau$, as we are interested in the time path of the state. For example, for the row involving transitions emanating from rating class A, $J_u$ is equal to 2 and $J_d$ is equal to 5.

Eta stands for a transition specific parameter and $\gamma$ is coined the rating class state sensitivity parameter. The subscript stands for the origin row and the superscript stands for an upgrade or a downgrade.

We solve by applying the concentrated likelihood technique (see Greene, 1993)[17]. Evaluating the actual second derivatives matrix of the concentrated log-likelihood function at the maximum likelihood estimates, delivers the standard errors[18].

There is actually no need to restrict the instantaneous failure rate state sensitivity to be identical among all transitions in the same direction. It is argued that the responses to the underlying modulating variables are different depending on whether the transition involved a change of issuer grade status in addition to the rating class transition[19]. Separating the effect of downgrades to speculative grade status from the ones to investment grade status allows us to test theoretical prescriptions about correlations with transition to rating classes closer to default. The following matrices that display the superscripts for the ’s illuminate this state sensitivity parameterization.

$$\left\langle \begin{array}{cccc|cccc} c & d_I & d_I & d_I & d_S & d_S & d_S & d_S \\ u_I & c & d_I & d_I & d_S & d_S & d_S & d_S \\ u_I & u_I & c & d_I & d_S & d_S & . & . \\ u_I & u_I & u_I & c & d_S & . & . & . \end{array} \right\rangle$$ Representing the structure of the first four rows, and

$$\left\langle \begin{array}{cccc|cccc} u_I & u_I & u_I & u_I & c & d_S & d_S & d_S \\ u_I & u_I & u_I & u_I & u_S & c & d_S & d_S \\ u_I & u_I & u_I & u_I & u_S & u_S & c & d_S \\ 0 & . & . & 0 & 0 & 0 & . & 0 \end{array} \right\rangle$$ the rows 5-8 of the intensity matrix, respectively.

The idea is that while retaining the distinction between investment grade and speculative grade issuer responses to the state variables, we elaborate as far as the transitions to the different categories are concerned. There is a different sensitivity of low-grade obligors regarding upgrade

[17] We discretize the integral and take first order conditions with respect to each $\eta$ and substitute the expression for $\eta$ in the log likelihood function and get an equation with one unknown, , after discarding irrelevant constants, not involving .

[18] Amemiya (1985) proves that the standard errors derived by the concentrated likelihood technique are identical to the ones calculated by the ‘brute force’ method.

[19] For example, the importance of the upgrade of Mexico’s sovereign debt to investment-grade status by Moody’s probably transcended the importance of the transition per se.

transitions to an investment grade rating class, relative to upgrade transitions to another speculative grade rating class. At the same time, there is a different sensitivity as far as downgrades are concerned. The corresponding structure is imposed on the investment grade section of the intensity matrix.

We now present the interpretation of the estimated state sensitivity coefficients. The derivative of the log upgrade downgrade ratio for each rating class with respect to the relevant state variables is the difference of the estimated coefficients for each destination transition group.

$$\log\left(\frac{\sum_{j}^{J_u} \lambda_{ij}^{u}}{\sum_{j}^{J_d} \lambda_{ij}^{d}}\right) = \log\left(\frac{\exp(\gamma_i^u X(t)) \sum_{j=1}^{J_u} \exp(\eta_{ij})}{\exp(\gamma_i^d X(t)) \sum_{j=1}^{J_d} \exp(\eta_{ij})}\right) \quad \text{Thus, } \frac{d \log(\frac{\lambda_i^u}{\lambda_i^d})}{dX_t} = \gamma_i^u - \gamma_i^d$$

For each rating class we calculate the ratio for the probabilities along the diagonal, for example for AA rated issuers we report the difference between the coefficients for upgrades and downgrades within investment grade[20]. We employ the parameterization that delivers different state sensitivity coefficients for each rating class, and the one that separates sensitivities within different grades when the intuition may be obscured. Significance tests based on the calculated standard errors are provided.

### b. Conditional non-parametric estimation and results

Proceeding with the investigation of the dependence of transition probabilities on economywide states we follow the spirit of Nickell et al (2000). We report Kaplan-Meier estimators of 1-month transition probabilities after dividing the 18-year sample in 3 categories, based on the higher, middle and lower third of the state variable realizations respectively. Subsequently, we compare these probabilities with the ones calculated from the whole sample and report the ones that are significant at a 5% level. The test statistic is based on the distribution

[20] According to financial industry practice, issues rated AAA, AA, A, BBB are termed investment grade and issues rated BB, B, and C are termed 'junk'. We use the terms speculative grade, high yield and junk bond, interchangeably. For AAA rated issuers we check the log odds of downgrade within investment grade versus downgrade to speculative grade.

of the difference between two multinomial variables under the null hypothesis of equality. The standard deviations of all elements of the transition probability matrix are calculated by equation (7a).

Each exhibit (tables III.b.1-2) depicts the difference between the relevant elements in the 'whole sample' and the subsample matrices, if statistically significant. These results are serving as a 'first pass' test regarding the relevance of estimating conditional transition probabilities and are overall encouraging and intuitive. To economize on space we will present the lower and upper third comparisons only.

As far as the credit state[21] is concerned the pattern is clear. A lower value for the credit state is associated with higher than average downgrade and lower than average upgrade probabilities. This relationship is reversed for higher credit state values but not clearly enough in economic significance terms. One can interpret this pattern as evidence of left skewness in credit risk, alternatively as an asymmetric response of creditworthiness to the economic cycle.

The same interpretation can be given to the results regarding stock market volatility. Especially interesting in both cases is the pronounced difference in the CCC to default transition probability between normal and 'recessionary' conditions. As far as the spot rate is concerned (not reported), the conclusions are not as clear but they are in the direction that is established and analyzed in the later sections.

## IV. The Variation of Credit Migration Risk

### 1. Credit Migration Risk Variation over the Business Cycle

#### a. Connection with Credit Risk Modeling Literature

[21] It is calculated as the first principal component (explaining around 2/3 of the variance) of the following three series, industrial production growth, new weighted average rating, and log upgrade downgrade ratio, which are motivated in the next section (Figure III.1.a.1). Although the 1981-82 recession was in all respects more severe than the 1990-91 one, the difference in the composition of the issuers' pool is manifested by the relative worsening of our measures in the 1990-91 period. In addition, the fall 1998 financial turbulence, not reflected in the evolution of the aggregate macro variables, is affecting the credit state measures.

In this section we examine the extent at which credit migration risk is correlated with market variables, with the main focus on interest rate and equity return measures. It is of importance to summarize the two approaches to credit risk modeling, the structural approach and the reduced form approach. In structural models starting from Merton, 1974, default is modeled as the 'first passage' time of the underlying asset process to the default boundary. This paradigm for corporate bond valuation admits a second generation of models where equity owners optimally trigger default when assets fall sufficiently low, e.g. Duffie and Lando, 1999.[22]

The reduced-form approach on the other hand, just postulates that default and rating migrations' intensities exist and may be dependent on state variables or other observables or unobservables (See for example Duffie and Singleton, 1999, Jarrow, Lando and Turnbull, 1997, Lando, 1998). This approach has lent itself extensively to the modeling of defaultable bonds and credit derivatives. Both have implications regarding the dependence of the credit event probabilities on market variables.

The Cox process formality put forward at the previous section can naturally accommodate this by making the cause specific instantaneous failure rates depend on market variables. The empirical evidence supporting this relationship is so far expressed as a dependence of credit spreads on the relevant variables. Duffee (1998) fits regressions of the form

$$\Delta Spread_t = b_0 + b_1 \Delta Y_{3m,t} + b_2 \Delta SL_t + e_t \tag{19}$$

The dependent variable, changes in the spreads of corporate bonds, is regressed on changes in the level and the slope of the Treasury term structure.[23] The estimated coefficients are negative and increase in absolute magnitude as credit quality decreases.[24] Based on this empirical work, we address similar effects on the *actual* credit event intensities, in contrast to the effect on the *risk-neutral* credit event intensities. We analyze the relationship between actual and risk neutral probabilities in Appendix B.

[22] They present a structural model that gives rise naturally to a reduced-form random intensity representation. They solve for a firm's capital structure and liquidation policy and derive the conditional distribution of the firm's assets given incomplete accounting information.

[23] The slope is the difference between the 10 and one-year constant maturity Treasury yields.

[24] See also Longstaff and Schwartz (1995).

As our focus consists of estimating conditional transition intensities, we can motivate our search for economically interesting factors by postulating a linear structure for the stochastic discount factor.

A one-factor Cox-Ingersoll-Ross model, for example derives from microeconomic foundations the short rate as the state of the economy. In general, as any term-structure model is equivalent to a time series model of the stochastic discount factor we start our exercise by conditioning on those states. Affine-yield models imply, in addition to the property that bond yields and hence bond prices are affine to the latent state variables, a linear structure for the SDF. Moreover, Litterman and Scheinkman (1991) perform a principal components analysis of the term structure and conclude that a high percentage of the variance can be attributed to three states that they connect to its level, its slope and its curvature. Consequently, trying to keep the connection with this literature we restrict ourselves to these obvious state variable candidates. In addition, we report sensitivities to the credit spread and credit market sentiment measures.

### b. Results

Proceeding with the results and their discussion, we address the issue whether one can interpret the sensitivities to be estimated as indicating causality confronting the issue of internal consistency of the estimation procedure. It is established that instantaneous failure rates are not identified when the conditioning variables are Granger-causing the hazard rates. Alternatively expressed, we have to allay concerns for possible feedback between the state variable process and the 'true' process that could be driving transition intensities as the perceived state variable could be driving the transition intensity process through its dependence on the common fundamental state. Consequently, this joint determination would render the estimation methodology untrustworthy.

The test statistics[25] for all the candidate state variables, except for the credit state, reported in table IV.1.b.1 fail to reject the null of non-Granger causality both for the investment and the speculative grade sections of the matrix. In any case, by lagging the state variables we assure that they are in the current information set, which for the time being we assume to coincide among the rating agency and the econometrician. Predictability in the statistical sense is being upheld.

[25] The testing methodology is presented in Appendix C.

We initially condition the instantaneous failure rates on spread information and the constituent series of the credit state, the growth rate of industrial production[26], and the new issue weighted average rating (NWAR)[27]. Every month several issuers tap the capital markets. Depending on the sentiment prevailing in the capital markets, issuers with specific characteristics are more prone to proceed and consequently the refinancing possibilities of lower-quality debtors are significantly hampered. During credit crunches only borrowers with stellar records can afford the increased scrutiny and withstand the concurrent turbulence. Therefore, the credit rating of new issuers is an accurate indicator of the quality of the issuers' pool in a clear countercyclical fashion. For example, during say the 1982 recession, only high-grade offerings could enter the market[28]. We use the spread between the yields on Baa Moody's rated issues and the 10-year constant maturity treasury (Table IV.1.b.2). [29, 30]

It turns out that as expected, downgrade intensities are positively correlated with an advantageous credit state, when we use the reduced form, and a low spread and the state sensitivity parameters for the credit state and the BBB rating class are statistically significant. There is an increased sensitivity to the credit state as we move down the investment grade quality ladder.[31] The correlation with the spread is more pronounced for the rating classes in the vicinity of BBB. A positive standard deviation change in the credit state and a negative standard deviation change in the spread are associated with a decrease in conditional downgrade

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[26] Available monthly, in contrast with the GDP figures that are available in a quarterly basis (Christiano, 1986).

[27] The weighted, according to distance covered and not to weighting with respect to the issuers' size, log upgrade-downgrade ratio possesses certain desirable properties that make it an appropriate measure of credit market conditions. Lonski (1999) claims that it closely tracks the developments in debt capital markets. Nevertheless, it effectively summarizes the rating activity, which we are trying to forecast (Kim (1999) falls into this trap). Granger causality tests fail for this series implying possible rating activity autocorrelation, which we address in the rating momentum section.

[28] The 'Weighted Average Rating' is determined by summing the products obtained by multiplying the principal amount of each underlying asset, including defaulted assets, by its Moody's rating factor and dividing this sum by the aggregate amount of all the securities in the relevant portfolio. (This practice is widely used to assign rating to CDOs). The rating factors used and their correspondence with S&P rating classes is the following:

| AAA | AA | A | BBB | BB | B | C | D |
|---|---|---|---|---|---|---|---|
| 1 | 23.3 | 123.3 | 410 | 1356.6 | 2810 | 6447 | 10000 |

[29] We report for every exercise the following parameters. Plus minus ceteris paribus standard deviations of the conditional downgrade probabilities are calculated along with log odds derivatives with respect to the state variables. We do not always present the estimates of the state sensitivity parameters and the respective standard errors because their information content is incorporated in the log odds derivatives calculations.

[30] In the appendix, we establish the connection between the spread and the *risk-neutral* default intensity.

[31] We note that the relative magnitude of the results should not be confused with relative economic significance, as a unit increase in the credit state is quantitatively more important than a unit decrease in the spread.

probabilities. Noteworthy is the fact that statistical significance for new issues weighted average rating is concentrated in the speculative grade rated issuers. Moody's numerical ratings increase exponentially as we move down the credit quality spectrum.

In the next parameterization, the intensities depend on the spot interest rate (Lando, 1998), for our purposes we use the 3-month interest rate, and the slope of the term structure. We use the difference between the 10-year and 1-year constant maturity Treasuries. In appendix D we report the stylized facts regarding the business cycle properties of the term structure to assist the interpretation of our findings.  We employ the status destination conditioning parameterization that allows variable state sensitivities among different rating classes.[32] The estimates (Table IV.1.b.3) indicate that overall, a high spot rate is correlated with increased downgrade probabilities, in agreement with the in-sample stylized facts. A relatively larger slope holding the short rate at its mean value, effectively an increase in the long rate, is overall mostly insignificantly associated with as expected higher downgrade probabilities. Reporting the log odds derivatives we are again in agreement with the aforementioned stylized facts. When the short rate goes up we are near the top of the cycle so forward looking relative upgrade transition intensities are responding negatively with similar conclusions holding for the long rate.  Only the insignificant association of relatively high CCC downgrade probabilities with a higher long rate is in discord with the expected result.

It turns out that the documented interest rate effect seems to run against the predictions that are implied by the aforementioned structural models. Both in the Merton (1974) and the Longstaff and Schwartz (1995) models, an increase in the default free rate is causing spreads to narrow as default risk goes down. In the Longstaff-Schwartz model, in the absence of recovery in case of default, the credit spread is derived in equilibrium as the risk neutral probability of default. In turn, this probability depends on the distance of the asset value of the firm from the default threshold. When the risk free rate goes up, the risk neutral drift of the asset process increases and the spread narrows. The empirical evidence of Duffee (1998) and Longstaff and Schwartz themselves is supportive of this conclusion.

Nevertheless, as Morris, Neal and Rolph (1998) argue, specifications that analyze the relationship between *changes* in the level of the term structure and *changes* in spreads focus on

[32] This being the only case where there exists relevant literature, we tolerate relatively large standard errors that are a byproduct of this 'overfitting', in order to be able to compare our results.

the short-run behavior and cannot address issues relating to the long run relationship. Further, letting the value of the firm itself be dependent on the risk free rate reverses the conclusions regarding the effect of an increase in the short rate.

Following their suggestion of a cointegration framework for the corporate and government bond rates[33], one can estimate an error-correction form for the two processes.

$$VEC = Baa_t - 1.23 Long10_t$$

This cointegrating relationship implies that the long run effect of a unit increase in the long Treasury rate is associated with an *increase* in the spread as the corporate rate increases by 1.23. Impulse response functions, clarify the documented short run negative response of spreads to increases in the Treasuries and the novel long run positive one.[34] The appropriate comparison with the downgrade correlation is the long run response of the spread as a rating transition is a reassessment of long run firm creditworthiness.

In order to judge the robustness of the results we redo the estimation by starting the sample in January 1983. It is argued that the behavior of interest rates exhibits a clear 'regime' shift discernible at that date. The estimated parameters do not display any particular difference (Table IV.1.b.4) and the signs and the significance of the log upgrade odds derivatives are identical with the full sample case. In addition, we examine the log odds derivatives restricting ourselves in Duffee's sample, January 1985 to March 1995. Some possible reconciliation can be discerned. It seems that for this sample, the log upgrade odds derivatives with respect to the short and long rates are positive for the AAA and AA rating class (AAA not reported) albeit insignificantly. On the contrary though, the sensitivity of the log upgrade odds with respect to a higher interest rate for the rating classes from A down is negative and increasing as we move down and statistically significant. Duffee (1999) restricts himself to the investment grade universe, although his correlations increase in the opposite direction as creditworthiness decreases.

[33] Regarding the wisdom of imposing cointegration on highly persistent, very possibly stationary processes for which unit root tests have low power against the alternative consult the references in Morris, Neal and Rolph (1998).
[34] It is very interesting to note in light of the criticisms fielded against the rating agencies, that whereas Morris et al find a delayed response of the spread to a change in the short Treasury rate, rating transition decisions respond instantaneously.

In the Litterman and Scheinkman (1991) framework the sources of risk that help describe the price risk inherent in default free bonds include a factor related to the curvature of the term structure. This curvature factor is proxied by changes in the volatility of the short rate.[35] Longstaff and Schwartz (1992) demonstrate that interest rate volatility has a significant effect on bond prices. Their findings indicate that the relationship between yields and volatility is negative as when interest rates become more uncertain investors increase their willingness to pay more for guaranteed returns.

Furthermore, interest rate volatility is even more important for the evolution of firm creditworthiness. Firm debt obligations have inherent put option and usually call option features embedded. The put option feature of corporate debt was laid down in the introduction to the previous section. In addition a big percentage of corporate bond issues has callability provisions. For example, one can expect the volatility stemming from the puttability to be more important as we are descending the credit quality spectrum (the option is more out-of-the-money as credit quality improves). On the contrary, given the interest rate an increase in interest rate volatility makes the call option purchased by the issuers of callable debt more valuable. These issuers are mostly investment grade.

We follow Longstaff and Schwartz (1992) in estimating a simple GARCH (1,1) in order to estimate the conditional volatility of the short rate. Including this estimated series in the instantaneous failure rate parameterization delivers the results in table IV.1.b.5.

Term structure curvature is economically and statistically significant for low-grade issuers. Relatively higher volatility is associated with lower downgrade probabilities across the board as the call options bought by the firm owners becomes more valuable. Nevertheless, statistical significance of the log odds derivative with respect to interest rate volatility obtains only for B rated issuers, with non-significant negative response for high-grade issuers. In the restricted specification, we obtain statistical and economic significance for the speculative grade section.

The aggregate default rate forecasting literature suggests that whereas nominal interest rates seem to explain little in the variation of annual default rates, *real* interest rates are significantly correlated with high default rates (Fridson, Garman and Wu, 1997). Notwithstanding our

[35] More accurately, the change in the volatility risk factor is usually interpreted as a curvature factor.

different focus and methodology we address this stylized fact by including the real interest rate as a conditioning variable.[36]

Table IV. 1.b.6 shows that downgrade probabilities are sensitive to the real interest rate in a statistically significant way. Furthermore, they respect its countercyclical properties as it is also manifested in the calculated semielasticities and the sensitivity to the real rate is more pronounced for low-grade issuers.

In the next series of experiments we examine the information embedded in equity returns. One can control for variation in actual default risk by including stock returns along with the spot rate as for being a relatively direct measure of the firms' prospects[37].

In order to sharpen the estimation of the equity return sensitivity parameters we restrict the coefficients to be the same among rating classes in the high and low-grade classifications and we apply the status destination conditioning specification. We want to guard against the following feature of rating activity as it relates to aggregate leverage measures. Especially the last decade, we have witnessed an explosion of stock buyback programs. It is usually the case that such an action is accompanied with an increase in the relevant stock price on the one hand, and by a simultaneous issuance of debt to finance the action. Therefore, it is often the case that a stock buyback announcement elicits a downgrade response from the rating agencies. Further, periods of high stock returns coincide with increased M&A activity that again trigger mostly downgrades by the agencies[38].

On the one hand, the correlations with respect to the level of the term structure information are broadly similar to the previous case. On the other hand, the relationship between stock returns and downgrade probabilities exhibits the expected pattern. A high contemporaneous stock return is associated with relatively low downgrade probabilities when significant, which is the case for the investment grade section (Table IV.1.b.7).

The higher and sharper correlation of high-yield debt issuers to the equity return, consistent with the equity-like features of low credit quality bonds established in the empirical literature (e.g. Shane, 1994), does not materialize yet.

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[36] We proxy the real rate by subtracting the year-on-year CPI growth rate from the 3 year constant maturity Treasury yield.

[37] The functional form that we estimate is close to the specification suggested by Jarrow and Turnbull (2000).

[38] It turns out that the percentage of rating actions as a response to M&A related events is high, especially in the 90's, see Lonski (1999).

Table IV.1.b.8 shows the effect of including another indirect measure of a structural influence on firm rating transition probabilities. Estimating a GARCH (1,1) for the equity return we extract a measure of equity return volatility, which serves as a proxy of the unobserved firm asset volatility. As expected, an increase in the volatility is associated with a statistically and economically significant increase in the downgrade probabilities. In addition, the correlation of the level of the volatility with the level of the downgrade probabilities is positive. Further, log odds derivatives agree with the aforementioned conclusions with statistical significance concentrated in the negative correlation between log upgrade downgrade probabilities of BBB and BB rated issuers and higher equity return volatility. Restricting the sensitivities within grades, we conclude that sensitivity to equity return volatility is more significant this time for the speculative grade section.

Another way to control for firm asset returns process features consists of examining whether zero-cost portfolios supposed to proxy for underlying state variables of special hedging concern to investors have any significance in modulating transition intensities. Motivated by the findings of Fama and French (1993)[39] we proceed by exploiting information incorporated in special equity return portfolios. We include the HML[40] (high minus low) portfolio in our parameterization, which is meant to mimic the risk factor in returns, related to book-to-market equity[41]. There exists a considerable controversy as to whether the 'value' premium is to be attributed to firm characteristics, ergo mispricing, or is a result of common nondiversifiable risk[42]. A natural interpretation is that during a credit crunch, liquidity crunch or flight-to-quality stocks in financial distress do very badly and this exacerbates the relative unattractiveness of stocks due to positive covariance with marginal utility. If this equilibrium interpretation truly holds one expects that returns on portfolios mimicking these risk factors enter significantly in the determination of downgrade probabilities.

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[39] They document for their government and corporate bond portfolios that whereas the term and default premia mimicking portfolios capture most of the variation in returns, for low grade corporate bonds their three stock market factors produce common return variation. When the term structure factors are included in the bond regressions, only for the low-grade corporates do the stock market factors retain their significance.

[40] We do not report results after including the SMB portfolio whose constituent companies mostly do not have any outstanding debt. Sensitivities are insignificant as it would be disturbing to find a significant effect on rating transition intensities of 'large' firms. In the 'financial accelerator' literature the link between firm size and external financial constraints seems to disappear when financial indicators, most importantly for our purposes whether a firm has a bond rating are taken into account (Whited, 1992).

[41] For information on their construction see Fama and French (1993).

[42] See, Daniel and Titman (1998).

(Table IV.1.b.9). Calculating the log odds derivatives with respect to the equity market related factors we corroborate their statistical significance in the intuitive direction. A higher return for the HML portfolio is associated with higher upgrade intensities. In the restricted parameterization, significance, both economic and statistical is retained for the investment grade section. For the unrestricted version, it is the case for the A and B rating classes. Another interesting feature of the estimation consists of the reduction in the magnitude and the significance of the spot rate sensitivities for the rating classes where the equity return factors are important. This is a facet of a result that escaped Duffee (1999). Overall we can support the claims of various authors that these zero-cost portfolios proxy for genuine macroeconomic risk.

## 2. In and out-of-sample forecasting

### a. Measures of fit

We need to suggest and defend ways of judging the extent that conditioning improves upon our forecasting ability. Statistical measures of specification analysis are well developed for univariate failure analysis, (see Lancaster 1990, Chapter 10) and quite developed but considerably harder for multivariate marked point processes (e.g. Russell and Engle, 1999) and are based on the unit exponential distribution of the integrated hazard. We propose *economically* interesting measures of fit based on relative value arguments.

We initially use a sum of absolute errors criterion relative to the Kaplan-Meier or the martingale benchmark. Nevertheless, this criterion has the obvious shortcoming that it treats for example a 0.1% error on the diagonal symmetrically with a 0.1% error on the AAA default probability. We propose to weigh the errors according to the realized incremental return of the bond that undergoes the transition.

Investors own corporate bonds in an effort to achieve returns over the corresponding ones offered by Treasuries of comparable durations. As Crabbe (1995) argues, over a horizon two factors can drive a wedge between yield spreads and realized incremental returns. Yield spreads may change and credit quality may change. Suppose initially that yield spreads are constant over time and the constant maturity AA spread is equal to 30-basis points.[43] If at the end of the year it gets upgraded to AAA where the spread is say 25 basis points the realized incremental return over Treasuries would be 38 bps.[44] If downgraded to BB it would get marked-to-market at a 130 bps spread and the incremental return would be –147 bps. Thus we suggest weighing the errors according to the absolute percentage change in the realized incremental return.[45]

The difference between the in and out-of-sample measures is that we use the estimated sensitivity parameters till the relevant date only, for the out-of-sample case. In both cases we use realized values for the conditioning variables[46]. Consequently, the out-of-sample forecasting exercise is not a joint test of the parameterization and the conditioning variables' forecasting model. In any case, our main focus consists of the 1-month ahead forecasting ability of the conditioning exercise. As we are using realized values of the state variables that belong to the current information set, forecasting of the state variable becomes irrelevant and we report 1-year ahead forecasts using the realized values as a check of the improvement or the deterioration imposed by the cumulative product.

### b. Results

We perform an in and out-of-sample comparison of the forecasting ability of the stochastic intensity parameterization, vis-à-vis the constant intensity, and the martingale ones. We describe the baseline strategy as follows.

For the in-sample backtesting exercise we will start from January 1990 and finish at December 1997. At each interval we compare the forecasts for the one-month and the one-year transition matrix provided by the exponential estimate, and the conditional transition matrix based on sensitivities estimated with data from the whole sample. In addition, we include the

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[43] A one-year mark-to-market horizon is used.

[44] $[\{ 1/(1+1.0025)^3 \} - \{ 1/(1+1.0030)^3 \} / \ 1/(1+1.0030)^3 ] - 1$

[45] When spreads change: Incremental return = Initial spread – Change in spread x Duration of portfolio in one year.

[46] Spreads can be forecasted by the change in the interest rates or by tacking or 'calibrating' risk premia on our estimated intensities. We use the realized ones from data kindly provided by Lehman Brothers.

forecast formed by a 'martingale' assumption. The comparison is made with the actual transition matrix.[47]

We first calculate the absolute error for each element of the matrix[48], i.e. we simply take the absolute value of the difference between the forecasted and the actual one-year transition probability matrix. We add the elements of each matrix up for each different method used for forecasting purposes. The mean sum of absolute errors (MSAE) for the calculated transition probability matrices constitutes our statistical criterion[49]. Our economic measure of fit justifies the use of overlapping periods to form consecutive forecasts in the 1-year horizon case, as every month contributes differently depending on the realized spreads.

Table IV.2.b.1 reports the mean sum of absolute errors for the exponential and the 'martingale' methods and compares them to failure rate parameterizations with respect to various parameterizations that include among others, the principal components of the Treasury term structure, the credit state and Baa spread one and the level and the slope of the Treasury term structure directly. We compare overall prediction accuracy and the ability to replicate the investment and speculative grade sections. Moreover, the suggested measure of economic fit is displayed.

The conditional forecasts are an improvement across the board both in statistical and economic terms. This conclusion holds both for the 'credit crunch' 1991-1993 period and the latter part of the decade. More specifically, the credit state parameterization exhibits an excellent performance during the credit crunch period, which is not continued in the rest of the testing period. This is clearly visible as far as the economic measure of fit is concerned, as it does not do well for the speculative grade section that includes many defaults. The performance of the parameterizations that take into account term structure information is quite robust. Nevertheless, performance in the speculative grade section is compromised in the 1994-1998 period, mainly because the restrictions on the coefficients do not allow separate sensitivities of CCC rated issuers, which adjacent to default, move in an idiosyncratic fashion as evidenced by their non-significant sign reversals of the log odds derivatives in the previous sections.

[47] The gradual elaboration being first in-sample estimated parameters using the actual state variable path, out-of-sample parameters using again the actual state variable path.

[48] Except for the last row, transitions out of default.

[49] This is close to the criterion employed by Kim (1999).

Although the methodology is not comparable, one may contrast these results with Kim's (1999) backtesting exercise. Taking aside the factors that make his forecasting exercise different and in a sense less representative of the true power of his method; he considers only quarterly transitions and uses the parameters estimated from the whole sample, it turns out that on average his conditional transition matrix is on average a worse fit than the unconditional one for the credit crunch period.

The out-of-sample exercise employs the winners from the in-sample horse race, namely level and slope of the term structure and equity volatility, credit state components and spot rate interest rate and equity return volatility parameterizations for the years 1997 and 1998.We condition on the realized values of the state variables that are part of the rolling information set (Table IV.2.b.2). Conditioning reduces forecasting errors for the overall CRTP matrix and the investment grade and speculative grade section. In addition, according to the economically motivated error metric, forecasting performance is still deemed better. It can be further commented upon that the investment grade section of the matrix is forecasted more accurately and this is reflected in the profit metric[50].

## V. Detecting Non-Markovian Behavior

### 1. Explicit Time Dependence

#### a. Motivation

In our previous intensity parameterizations, all action in the intensity front is captured by state dependence. We argue that time effects entering independently of the state are important. As has already been established in Moody's Investors Service own research, rating transitions display a certain degree of 'accelerated failure'.

Carty and Fons (1993) fit a Weibull hazard specification, without taking into account the 'competing risks' features of failure. Their distributional assumptions do not allow for a flexible specification for the relative rating action risk. Further they restrict the scale factor to be identical

[50] Weighting the rows according to the relative actual debt outstanding for each rating class can 'reward' the better performance as far as investment-grade issuers is concerned.

among rows. They report a scale factor marginally greater than one for AAA, AA and A rating classes, which falls to less than one as we progress to the speculative grade categories. As credit quality decreases the hazard rate function changes from increasing to decreasing. The explanation offered consists of the observations that for high investment grade issuers there is nowhere to go but down which causes the gradual erosion of their creditworthiness. At the same time, as far as low-grade issuers are concerned, initially rating activity is high, but as time goes by and the issuer has not defaulted creditworthiness tends to level off. Thus, they do not distinguish between possible upgrades or downgrades and they subsume genuine state dependence under ‘plain vanilla’ time dependence.

### b. Estimation Methodology

#### i. ‘Vanilla’ Time Dependence

In order to compare our work with the literature we estimate a functional form for the cause-specific failure rate that does not allow for the distinction between state and time dependence.

We should reflect on the substantial difference of focus between our approach and the ‘aging’ one put forward by Altman (for a comparison among the various approaches, see Altman 1997). Altman’s (e.g. Altman & Kao 1991,1992) studies document the existence of the following pattern of rating activity. If one examines a pool of corporate bonds and their accompanying rating from the date of the original issuance up to ten years post-issuance, the following pattern of rating activity is encountered. An ‘aging’ effect manifests itself as the greater short-term tendency of older bonds to be up- or downgraded relative to newly issued bonds, in Altman’s words. Moreover, if we restrict ourselves to the ultimate credit event of default, the pattern becomes one of hump-shaped default intensity, the intuition of this result being that a company with a newly issued bond has at least received its face value. Consequently, it obtained the liquidity to service its debt obligations, among other things. Further, Altman claims that most bonds or loans are not subject to credit review during their first year.

Our approach is not motivated by the same considerations, although it may be possible to capture related phenomena. The database that we use does not contain information on a specific issue from its time of issuance till either the end of the sample or possible default or withdrawal.

We assess the senior bond equivalent of each issuer, regardless of the issuer's size or the number of issues outstanding. The comparable work in our context regarding potential time dependence in rating activity consists of Carty and Fons (1993).

An important aspect of our exercise consists of examining whether rating activity is time-dependent thus invalidating the Markov assumption without appropriately expanding the state with the cost of additional complexity. The exponential distribution (Weibull distribution with shape parameter 1.0) is the key assumption for the Markovian property, which is crucial for the results derived in the reduced form intensity literature of pricing defaultable debt and credit sensitive derivative securities.[51]

The cause-specific instantaneous failure rate distinguishes now between the failure acceleration coefficients for downgrades relative to upgrades whereas the scale parameter is allowed to differ among every destination. This insight allows for increasing downgrade and decreasing upgrade probability over time for different rating classes. We note that the previous case is nested in the current specification when $\alpha=\beta=1$.

The upgrade and downgrade hazard rates and the likelihood function are (Equations 21a,b,22)

$$\lambda_{ij} = \alpha_i t^{\alpha_i - 1}\eta_{ij}\,,\ \lambda_{ij} = \beta_i t^{\beta_i - 1}\eta_{ij}$$

$$L = \langle \prod_{j=1}^{J_u} \{ \prod_{i=1}^{N_j} \alpha t_i^{\alpha-1}\eta_j \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1}\eta_j + \sum_{j=1}^{J_d} \beta u_i^{\beta-1}\eta_j \} du)\}\rangle \times$$

$$\langle \prod_{j=1}^{J_d} \{ \prod_{i=1}^{N_j} \beta t_i^{\beta-1}\eta_j \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1}\eta_j + \sum_{j=1}^{J_d} \beta u_i^{\beta-1}\eta_j \} du)\}\rangle \prod_{i=1}^{N_c} \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1}\eta_j + \sum_{j=1}^{J_d} \beta u_i^{\beta-1}\eta_j \} du)$$

The estimation results[52] are presented in table V.1.b.i.1.

Upgrade intensities are characterized by an increasing hazard rate. The increasing pattern becomes more pronounced as we move towards the lower end of investment grade issues. Nevertheless, the upgrade intensity of speculative grade bonds, although increasing with time, breaks the progression. Moreover, as we move down the credit quality ladder the downgrade

[51] This is true for the time-inhomogeneous Markov chain approach, where we condition on state variables.

[52] The concentrated likelihood technique reduces the number of unknowns to the upgrade and downgrade acceleration parameters and .

intensities are becoming less increasing over time. Importantly though, the downgrade intensity of a CCC rated issuer is decreasing over time. Estimates are statistically significant.

We conclude that the results of Carty and Fons are not particularly robust with regard to the rating activity concerning investment grade bonds, as a consequence of their identical treatment of upgrades and downgrades. However, their rationale for the behavior of low-grade issuers survives in our framework. If low-grade issuers survive the starting crucial time period, it means that they have escaped default, being the only downgrade option, identifying them as more creditworthy securities than initially perceived.

Consequently, we have highlighted important potential non-Markovianities in the behavior of rating activity. The next step is to integrate state and time dependence, in an attempt to examine whether these features survive the introduction of economywide state variables.

### ii. State and Time Dependence

The cause-specific instantaneous rate of failure for the stochastic time inhomogeneous case and the corresponding likelihood function take the following form[53]. *Equations (23a,b,24a)*[54]

$$\lambda_{ij}(X(t)) = \alpha_i t^{\alpha_i - 1} \exp(\eta_{ij} + \gamma^u{}_i X(t)),\ \lambda_{ij}(X(t)) = \beta_i t^{\beta_i - 1} \exp(\eta_{ij} + \gamma^d{}_i X(t))$$

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[53] We exponentiate the destination-specific scale parameter for identifiability reasons.

[54] One should take particular care with regard to the integral discretization performed. Whereas we assume that the states are month-by-month piecewise constant, the integral for each interval should be repeatedly evaluated. The generic term is

$$\int_\tau^t \exp(\gamma X(u)) \alpha u^{\alpha-1} du$$

The limits of integration are the calendar dates of the start and the finish of the time of each issuer in the particular rating class. Nevertheless, whereas the exponential part of the expression is evaluated at the relevant calendar date value, the integral is solved for the elapsed time. The expression becomes

$$\exp(\gamma X(t_1)) \int_0^1 \alpha u^{\alpha-1} du + \exp(\gamma X(t_2)) \int_1^2 \alpha u^{\alpha-1} du + \ldots + \exp(\gamma X(t_n)) \int_{n-1}^n \alpha u^{\alpha-1} du =$$

$$= \exp(\gamma X(t_1))(1^\alpha - 0^\alpha) + \exp(\gamma X(t_2))(2^\alpha - 1^\alpha) + \ldots + \exp(\gamma X(t_n))(T^\alpha - (T-1)^\alpha) =$$

$$= \sum_{t=\tau}^{T_i} \exp(\gamma X(t))(t_i{}^\alpha - t_{i-1}{}^\alpha)$$

The lifetime of a stay at a rating class is = t1 < t2 < …< tn-1< tn = T.

$$L = \langle \prod_{j=1}^{J_u} \{ \prod_{i=1}^{N_j} \alpha t_i^{\alpha-1} \exp(\eta_j + \gamma^u X(t_i)) \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \beta u_i^{\beta-1} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle \times$$

$$\langle \prod_{j=1}^{J_d} \{ \prod_{i=1}^{N_j} \beta t_i^{\beta-1} \exp(\eta_j + \gamma^d X(t_i)) \exp(-\int_{\tau_i}^{t_i} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \beta u_i^{\beta-1} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle \times$$

$$\prod_{i=1}^{N_c} \exp(-\int_{\tau_i}^{t} \{ \sum_{j=1}^{J_u} \alpha u_i^{\alpha-1} \exp(\eta_j + \gamma^u X(u)) + \sum_{j=1}^{J_d} \beta u_i^{\beta-1} \exp(\eta_j + \gamma^d X(u)) \} du) \} \rangle$$

The results using the 3-month interest rate and the slope of the term structure as conditioning variables are presented in table V.1.b.ii.1. We only present the upgrade and downgrade acceleration coefficients and their standard errors. In figure V.1.b.ii.1, we display first the cross section of the downgrade probabilities given that a rating action takes place for B and CCC rated issuers. Given the magnitudes of the acceleration coefficients the ‘term structure’ of downgrade probabilities is downward sloping. Figure V.1.b.ii.2 presents these probabilities when we are not conditioning on the rating event. For all rating classes except for CCC, both upgrades and downgrades display increasing acceleration. Therefore the negative slope for CCC rated issuers follows.

Table V.1.b.ii.1 displays the semielasticities of the conditional on rating action downgrade probabilities for an issuer that has duration of one month and one year respectively in the particular credit quality class. What stands out is the fact that even though the relative directions remain intact after taking into account time dependence, the sensitivities are much lower for recently transitioned bonds.

### 2. Rating Momentum

Another aspect of the credit rating assignment process that has received attention in the context of modeling CRTP matrices in terms of its implications for its statistical characterization is the dependence of the current rating on the previous one. Empirical studies such as Carty and Fons (1993) document that prior rating changes may carry predictive power for the direction of future rating changes. This property is called rating momentum. It implies that a firm being upgraded (downgraded) is more susceptible to subsequently being further upgraded (downgraded). Therefore, the Markov property fails, as the current rating does not fully determine the transition intensities.

Carty and Fons (1993), test separately the hypotheses of upward and downward rating momentum. They conclude that there is no evidence for the existence of upward rating momentum, whereas on the contrary, as far as downgrades are concerned they cannot reject the null of downward rating momentum. This attempt does not fit in a broader estimation paradigm. Moreover, accounting only for transitions over a yearly horizon, probably underestimates momentum, as the transitions that happen sequentially in response to the same news are not spaced far apart.

Our approach is close in spirit to the study of ‘stigma’ in unemployment studies (e.g. Heckman and Borjas, 1980). Upgrade (downgrade) transition intensities become,

$$\lambda^{u}{}_{ij} = \exp(\eta_{ij} + \gamma PU_i), \lambda^{d}{}_{ij} = \exp(\eta_{ij} + \delta PD_i) \tag{26}$$

PU and PD are dummies that take the value of one when an upgrade (downgrade) has been preceded by a rating action in the same direction, and zero otherwise.

The existence of both upward and downward positive rating momentum cannot be rejected. (Table V.2.1) After establishing that there is prima facie evidence regarding the importance of rating momentum, we again examine whether survives the inclusion of economywide state variables. We use the spot interest rate and present the results in table VII.2b.

Both upward and downward rating momentum survives the presence of a state variable in a statistical significant way. The calculated semielasticities of the downgrade probabilities with respect to the nominal interest rate are in the expected direction, in the sense that the increase in the relative downgrade probability given that a rating action happens, is more muted for the issuers that possess downgrading momentum (Table V.2.2).

This evidence alerts researchers and practitioners of modeling the credit rating process as Markov without appropriately expanding the state space.

### 3. Alternative Methods for Identifying non-Markovian Behavior: A semiparametric estimator

In the literature for estimating multiple spell, multiple state models, a specific type of methodology that relies on the separation between duration and state transition related parameters is proposed. Heckman and Singer (1984) observe that one can proceed by breaking his estimation into two components. On the one hand, the probability of transition to a specific state given that a transition happens will be estimated using data on every possible origin and destination pair. On the other hand, the process governing the timing between events will be estimated using standard survival methods. As they report, the two estimators are consistent, asymptotically normal and efficient, and independent of each other as long as the number of firms sampled becomes large. We have already encountered this separation in the exponential parameterization, where the intensity estimator can be expressed as the conditional transition probability times the inverse of the (constant) waiting time, the expected time spent in the relevant rating class.

Thus, we can leave the exact specification of the transition process unspecified or as the solution for the embedded Markov chain, which naturally translates into the ratio of the actual transitions over the potential ones for each rating class. We are not delving into this issue anymore as it is adequately covered in the previous sections.

We model the duration of stay at each rating class using proportional hazard models. As our specification is motivated by the effect of time varying covariates we assume that they are the same functions for each member of the particular rating class. The proportional hazard parameterization is used extensively in the literature and importantly allows for the calculation of log odds derivatives with respect to the time varying covariates that are independent of time making the results directly comparable with the previous sections. The instantaneous upgrade or downgrade hazard rate for each rating class has the following general form,

$$\theta(x,t) = k_1(x(t))k_2(t) \tag{30}$$

The $k_1$ function is called the baseline hazard rate and is also assumed common for each member of the population. We take into account several ways of parametrizing the baseline hazard and also leaving it unspecified. By analyzing several forms of time dependence and incorporating rating momentum[55] at the same time we can examine the robustness of our previous results in a fully non-Markovian setting.

[55] This section deals with momentum in a more thorough fashion in the sense that it takes into account *potential* momentum that did not materialize. Hence, momentum manifestations in this section are even more significant as its

For every rating class i, whether an upgrade (u), or downgrade (d), we estimate the following objects,

$$\theta_{i,u}(x,t) = k_{i,1u}(x(t))k_{i,2u}(t), where$$
$$k_{i,1u}(x(t)) = \exp(\beta_{i,u0} + \beta_{i,u1}spot(t) + \beta_{i,u2}slope(t) + \beta_{i,u3}garch(t) + \beta_{i,u4}vol(t) + \beta_{i,u5}momentum)$$

Subsequently, we calculate the derivative with respect to each state variable for the log upgrade downgrade hazard ratio,

$$\log\{\frac{\vartheta_{i,u}(x,t)}{\vartheta_{i,d}(x,t)}\} = \log\{\frac{k_{i,u}(x(t))}{k_{i,d}(x(t))}\} + g(t), thus$$
$$\frac{\partial \log\{\frac{\vartheta_{i,u}(x,t)}{\vartheta_{i,d}(x,t)}\}}{\partial x_k} = \beta_{i,uk} - \beta_{i,dk} \tag{31}$$

We now report and analyze the different functional forms we use for the baseline hazard and their characteristics in addition to the preservation of the proportionality property. We have already used and presented the exponential case that does not exhibit duration dependence, and the Weibull distribution that exhibits monotone duration dependence. In addition, we examine the Gompertz distribution that allows for exponentially increasing or decreasing with time hazard rates. This is specified as,

$$\vartheta_{i,u}(x,t) = k_{i,u}(X(t))\exp(\gamma t) \tag{32}$$

Thus depending upon the sign of gamma we have exponentially increasing or decreasing hazard rates. Further, we employ the Cox (1972) semiparametric methodology that allows for separate estimation of the elements of k without specifying the baseline hazard, which can be recovered at a second stage.[56]

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importance is not overstated as in section 2. Employing this momentum definition in the methodology of the previous section renders the estimates of the investment grade section very unstable.

[56] For its motivation as a partial likelihood estimator, the derivation of the likelihood function and the backing out of the baseline hazards see Kalbfleisch and Prentice, 1980.

We present the log odds derivatives and their statistical significance at a 5% level for each rating class from AA down for the 4 different parametrizations. We do not present all the parameter coefficients with robust standard errors that take into account that independent durations for the purposes of estimation come from the same firm for economizing on space. In any case, estimates and standard errors are used to derive the aforementioned measures (Table V.3.1). Momentum sensitivities are presented when they are significant.[57] (Table V.3.2)

The following comments are in order. First, there are no sign reversals for log odds derivatives as far as different parameterizations are concerned. Comparing the results with the previous sections, we can report that overall higher short and long rates are associated with higher relative downgrade probabilities, except for the AA rating class. The sensitivity is getting stronger as we move down the credit quality spectrum. Equity volatility sensitivities are mostly insignificant and unfortunately of the 'wrong' sign when significant. Similarly, we run into problems with the accord of the interest rate volatility sensitivities with the previous results. We venture the following interpretation for the discrepancies. The results in the previous sections are based on the parameterization that employs status destination conditioning and a state specific baseline failure rate. For associations that are hard to pin down it is advisable to trust the previous results that isolate the sections of the matrix that are less dependent on transitions that may obscure the 'true' picture.

Nevertheless, the methodology is ideally suited to examine the relative importance of rating momentum in the presence of state variables and time dependence. It is interesting to note that for whatever rating class and direction momentum is significant it remains so for any parameterization. In any case we report the estimates for the momentum coefficient under the Weibull hazard rate assumption. Downward momentum is significant for the speculative grade rating classes, which is in accordance with common practitioners' wisdom among others. On the contrary, upward momentum all but disappears. The results of Carty and Fons (1993) resurface after all. Interestingly enough, a property of the rating process that we tentatively coin 'rating mean reversion' emerges. It seems that a previous upgrade actually reduces your upgrade hazard rate for BBB and A rated issuers. In essence, there is rating threshold that is very hard to surpass

[57] They are nevertheless taken into account each estimation regardless.

or alternatively that there is an 'average' rating that is to be attained for the majority of firms under normal circumstances.

We continue with an analysis of the choice between alternative competing characterizations of the baseline hazards. Aside from the proportional hazard models we can estimate accelerated failure time models. The name implies the time deformation property that is their distinguishing characteristic. They mostly do not allow the functional separation between time and covariates but they can exhibit non-monotonic hazards. We examine log-logistic hazards with

$$\theta(x(t),t)=\frac{k_1(x(t))\frac{1}{\gamma}t^{\frac{1}{\gamma}-1}}{1+k_1(x(t))t^{\frac{1}{\gamma}}} \tag{33}$$

If gamma is less than one, theta increases from zero to a single maximum and then approaches zero as t tends to infinity. In order to distinguish between those competing models we resort to the Akaike Information Criterion, that penalizes every estimation according to the number of ancillary parameters estimated[58] (Table V.3.3). Hence, the AIC for our purposes is[59]

AIC = -2logL + 2(p + 1)

The results are mostly consistent over the credit quality spectrum. After taking rating momentum into account for all parameterizations and rating classes the thrust of the results derived in the previous sections survive. The Weibull time acceleration pattern is the norm for all rating classes except for the CCC downgrades, which becomes intuitively decelerating. Even though the non-monotonic version of the log-logistic hazard is the winner in the AIC horserace we cannot the support this pattern with an Altman-like intuitive story. At we analyze issuer-based ratings and not issue based ones and we are dealing with upgrades and downgrades and not defaults, the aforementioned story of default hazards being initially flat, then increasing and finally decreasing as we approach maturity can probably not be applied in our setting but only in

[58] I.e. time acceleration parameter for the Weibull, gamma for log-logistic.
[59] Whenever the generalized gamma hazard rate is well behaved we can perform another simple test for the validity of the Weibull as it is nested within it.

the CCC downgrade case where the evidence in favor of the log-logistic is overwhelming. Moreover, even though the hazards produced by the Gompertz parameterization seem the ones closest to the nonparametric Moore and Pyke (1968) calculations[60], they are not succeeding according to the AIC and even though insignificantly the increasing hazard rate is reversed occasionally.

## VI. Observed and Unobserved Heterogeneity

### 1. Observed Heterogeneity

So far in our discussion we have been focusing on analysis and results on the observed or 'aggregate' hazard rate. Whereas we state unambiguously that it truly is the correct object of interest for the purposes that our study is a legitimate starting point, we attempt to address the issue of whether observed or unobserved heterogeneity among issuers can be found to play a role in determining failure rates. Let us reiterate that this can be thought as a non-issue as we are trying to deal with exactly how economywide state variables would affect the 'average' issuer, which implies that the 'worse' issuers would be more sensitive and correspondingly for the 'better' issuers. Nondiversifiable credit risk, i.e. not due to dependence on state variables would by definition 'average' out in a large sample of firms. If we would have been dealing with a corporate default model we could legitimately try to account for possible differences among firms. In essence, the implicit assumption in the study so far is that the rating agency is doing a decent job in categorizing firms in fairly homogeneous buckets regarding credit risk. Nevertheless, the rating agency undoubtedly possesses information unavailable to the econometrician that we are obliged to take into account into our estimation.

Dealing with time dependence and rating momentum we are already attempting to introduce *observable* heterogeneity into our work. Another obvious characteristic that may clearly be dealt with is industry participation. S&P categorizes the rated issuers into industrials, utilities and

[60] Not reported.

financials.[61, 62] We initially examine the associations of the usual candidate state variables with the instantaneous failure rates without exploiting any firm-specific information. The experiment we undertake is designed to examine the sensitivity of the different industries to the conditioning state variables. Sectors of the economy have probably different recession sensitivity. We present estimation results using the level of the term structure and the S&P 500 return volatility as the states. We separate the transition history sample according to industry and perform the usual procedure. From the 12076 independent transitions, censored or not, there are 1852, 2783 and 7441 observations classified as utilities, financials and industrials respectively. Tables VI.1.1, 2, 3 contain the results.

We use the direction conditioning characterization, restricting the sensitivities within high and low-grade rating classifications respectively. We get the expected results for both state variables, that is a relatively low spot rate is associated with lower downgrade probabilities except for high-grade issuers classified as utilities. We revert to the subject of interest rate sensitivity of utilities after we incorporate time dependence and momentum. Overall an increase in the spot rate is associated with a decrease in the upgrade downgrade log odds as is an increase in equity return volatility.

It is clear that financials are more sensitive to the term structure factor. It should be further noted, that there are quite few observation of junk bond issuance or traversing by issuers classified as financials. A junk bond rating for a financial institution is usually disastrous.

Expanding our focus in the spirit of the previous section, we introduce time dependence in the context of the proportional hazard Cox (1972) model, where the baseline hazard is left parametrically unspecified.

$$\vartheta_{i,u}(x,t) = k_{i,u}(X(t))k_{0,u}(t), \vartheta_{i,d}(x,t) = k_{i,d}(X(t))k_{0,d}(t)$$

[61] The relative weight of each sector among the rated universe provides among others a telling narrative of the important forces shaping the US economy over time. As various graphs in Moody's rating migration study (1997) illustrate, the cross-section of US bond issuers has moved in tandem with the patterns in the capital formation process. For example, railways comprised the bulk of issues in the beginning of the century, but consolidation in transports and utilities and expansion in industrial issues have reversed that.

[62] We revert to the use of conditioning on economywide state variables instantaneous failure rates for different industry groups when analyzing the calculation of correlations in the next section.

We report the log odds derivatives with respect to the state variables after taking into account rating momentum as well (Table VI.1.4). The following comments are in order. On the one hand, the sensitivity of issuers categorized in financials stands out in a statistically significant fashion as well. On the other hand, if we believe the business cycle story about the overall interest rate sensitivity, utilities manifest themselves as a good credit hedge[63] for the short rate. Hence, we conclude that observable heterogeneity in the form of industry participation allows for both statistical and economic sharpening, even though the sample shrinks.

One can proceed in another fashion if one does not believe that economic forces affect industries differently. This approach is closer in spirit to the previous section, where now we employ stratified by industry partial likelihood. Thus, for industry j,

$$\vartheta_{i,u,j}(x,t) = k_{i,u,j}(X(t))k_{0,u}(t), \vartheta_{i,d,j}(x,t) = k_{i,d,j}(X(t))k_{0,d}(t)$$

Hence, we allow the heterogeneity to manifest itself in the baseline hazard, whereas we restrict the coefficients to be equal. The results (Table VI.1.5) should be directly comparable with the Cox parameterization in the previous section and it turns out they are. The major difference consists of the reduction of the A and BBB interest rate sensitivities, which is translated as baseline hazard for financials that is increasing faster. Thus, this parameterization leads one to interpret this higher interest rate sensitivity as higher level of scrutiny by the rating agency that changes its credit indicators faster and these periods of rating activity happen to coincide with higher interest rates. Nevertheless, this is not correct, as we allow for different baseline hazards without restricting coefficients in the previous setting.

### 2. Unobserved Heterogeneity

Flinn and Heckman (1982) define unobservable heterogeneity as unmeasured exogenous variables that differ among individuals (firms) and that may differ over time for the same individual (firm), reserving the term for unobservables as perceived by the econometrician. Nevertheless, by Barlow and Proschan (1965) as our fitted hazards exhibit mostly positive

[63] Which is the common wisdom in the stock market.

duration dependencies, at least some firm exit distributions should exhibit positive duration dependence.

Suppose all firm variation in the hazard rate function can be characterized by time varying observed explanatory variables x(t) and an unobserved heterogeneity term v.[64] The standard mixed proportional hazard model defines the firm instantaneous failure rate as

$$\vartheta(t \mid x, v) = \psi(t)\vartheta(t)v$$

If we do not condition on unobserved heterogeneity and by applying Laplace transformations or the Cauchy-Schwartz inequality one can derive that

$$\frac{d \log \vartheta(t \mid x)}{dt} = \frac{\psi'(t)}{\psi(t)} - \frac{Var(v \mid T > t, x)}{E(v \mid T > t, x)} \psi(t)\vartheta_0(x)$$

Meaning that in the presence of non-degenerate unobservable heterogeneity, ‘weeding out’ of the firms with the highest values of v, induces a more negative duration dependence in the observed hazard rate than the ‘true’ underlying one. Moreover, unaccounted for v, biases hazard semielasticity with respect to the state variable towards zero.[65]

$$\frac{d \log \vartheta(t \mid x)}{dx} = \frac{\theta_0'(t)}{\theta_0(t)} - \frac{Var(v \mid T > t, x)}{E(v \mid T > t, x)} (\int_0^t \psi(\tau) d\tau)\vartheta_0'(x)$$

Undoubtedly, these possible departures from the underlying tenuous assumption of identical information sets between the econometrician and the rating agency or alternatively that the rating agency may have more information than the researcher but does not act upon it, oblige us to attempt to address its importance in our data.

Our first effort to examine the effect of unobserved heterogeneity consists of reintroducing information that we discarded in the previous sections for legitimate reasons. Our dataset

[64] We follow van den Berg’s (2000) discussion.

[65] Which does not imply under-estimation of x’s effect on the individual hazard as van den Berg (2000) stresses, as the sensitivities are jointly determined.

includes information about credit rating transitions between modified ratings.[66] In the interest of robustness in the forecasting exercise we consolidated those in the 8 rating classes, including default that we have dealt with so far. Undoubtedly, by performing the previous calculations with the restricted sample we were subject to the criticism of neglecting heterogeneity that turns out to be observable.

For example, within the AA rating class, there are creditworthier, AA+, average, AA, and less creditworthy, AA-, issuers, whose transitions can be triggered by or associated with different state variable patterns. Hence, an intuitive test is the comparison of the estimated sensitivity parameters under the status destination conditioning characterization under the alternative information sets.

It turns out that the estimated log upgrade odds derivatives do not differ in a statistically and economically significant fashion from the ones calculated neglecting the extra information. The signs are remaining the same and we cannot reject the hypothesis of equality. In tables VI.1.1 and 2 we report the estimates for the alternative information setups. Only in the log upgrade to low grade/downgrade odds derivatives with respect to the interest rate level and its volatility do we encounter a statistically significant difference. The direction of the sensitivities remains intact.

Moreover, we suggest examining the importance of unobservable heterogeneity by appealing to discrete choice theory. We can conclude from the previous section that even when adjusting the pattern of time dependence the state variable sensitivities remain similar. Thus, our focus is now to focus on rating class-specific unobservables that influence the covariate effects.

Assuming J possible states, a.k.a. rating classes, I firms, a vector of time varying exogenous variables $Z_t$, and a common among all firms unobservable factor f, we have the following decision rule governing observed transition patterns between states.[67] The probability that state k is chosen, *conditional* on Z and f is given by the probability that the utility of state k for firm I is larger than all other states and it involves a multiple integral of the cumulative error following a Type I extreme value distribution. In this case we write the familiar multinomial logit form for this conditional choice probability.

[66] The modified ratings are the following: AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, BB-, B+, B, B-, CCC, D, NR.

[67] In this discussion we adapt arguments put forward in the documentation of Son-of-CTM program provided courtesy of Professor Heckman.

$$P_{kt}(Z_t, f_i, parameters) = \frac{\exp(\beta_{k0} + Z'_{tk}\beta_k + \alpha_k f_i)}{\sum_{j=1}^{J}\exp(\beta_{j0} + Z'_{tj}\beta_j + \alpha_j f_i)}$$

It is assumed that the elements of $f_i$ are independently distributed random variables. The density functions associated with each element of f are approximated by discrete distributions. Consequently, the likelihood function integrates out this unobserved variation. We assume one dimension of unobserved heterogeneity with five points of support. The factor-loading alpha is estimated and its significance displayed along with the relative location of the points of the distribution support and their associated probability masses. Hence, we can judge the extent at which unobserved heterogeneity can be found to be important.

It seems that even though unobservable heterogeneity of the characteristics of firms turns out to be statistically significant the direction of the odds of any particular rating class being chosen in a particular economic environment as being proxied by the spot rate are not substantially affected especially in terms of their direction. Specifically, estimating the model without any heterogeneity does not result in log odds derivatives of a different sign or substantially different magnitude than the ones calculated in its presence. In addition, state specific heterogeneity coefficients are quite insignificant and the heterogeneity distribution quite unstable (Tables VI.2.1-2). The distribution is left skewed and interestingly enough displays the inverse of the credit loss distribution pattern, which is right skewed; a relatively certain coupon, punctuated by the relatively rare default.

## VII. Conclusion

In this work we suggest an alternative method of deriving nonparametric credit rating transition probability matrices. Recognizing the shortcomings of not imposing some structure and not taking into account state dependence we develop a methodology designed to allow conditioning information of credit market institutional features and of various market variables.

We undertake a study of the variation of credit migration (which includes default risk) over the business cycle. We report that high short and long nominal and real short interest rates, high equity return volatility, low equity returns are generally associated with high conditional downgrade probabilities. The improvement in CRTP matrix forecasting accuracy by utilizing state variable information is overall decent, both statistically and economically, in in-sample and out-of-sample experiments. The importance, both statistical and economic, of the term structure and the equity return variables, allows an interpretation that can defuse some of the criticisms directed at the rating agencies. They have been the targets of critical comments relating to the forward-looking nature of their rating activity. The fact that ratings are contemporaneously correlated with market variables in the anticipated pattern goes a long way towards addressing this criticism.

Our next contribution consists of introducing issuer-specific information in the form of time dependence and rating momentum. We examine whether the information provided by the state variables is subsumed or is augmented by incorporating various parametric or nonparametric types of time and occurrence dependence in the credit rating transition process. The overall pattern of correlations remains mostly intact, although non-Markovian behavior of the rating process is significant. Examining the effect of taking into consideration industry classification reveals the different sensitivity of certain sectors to term structure variables with multifaceted implications.

In addition we examine the importance of unobservable heterogeneity, i.e. the importance of rating agency information that is not available to the econometrician and how it may alter the conclusions about the direction of state variable associations. On the one hand, we integrate the heterogeneity distribution out of the likelihood function. On the other hand we examine the impact of reincorporating heterogeneity information that we discarded in the interest of robustness in accordance with the literature. We conclude that unobservable to the econometrician heterogeneity does not alter the conclusions in an appreciable fashion.

Our estimation strategy lends itself naturally to modeling of credit event correlations among obligors arising from the dependence on common factors. Thus, we take into account the interaction between credit and market risk.

# Appendix A

## Data Description

Credit ratings are published in a timely manner by rating agencies and provide investors with invaluable information about firms', sovereigns' and municipal authorities' abilities to meet their debt obligations. Standard & Poor's is a leading bureau specializing in authoritative information regarding various credit events. This S&P proprietary database consists of ratings and defaults for industrial and transportation companies, utilities and financial institutions on senior unsecured long-term debt. As we prefer to concentrate on corporate bonds, sovereigns, municipal debt issuers, structured finance transactions, private placements and issuers with only short-term debt are excluded. In total, the credit experience of numerous issuers that sold long-term debt publicly between January 1$^{st}$ 1981 and December 31$^{st}$ 1998 is thoroughly documented.

In addition, we exclude non-US issuers. Whereas virtually all debt issuance by US-based firms is subject to the agencies' scrutiny there is a prominent selection bias regarding non US-based issuers. Moreover, in the conditioning exercise we want to address common state dependencies. Notwithstanding increasing global macroeconomic interdependence, it is hard to construct and defend common states of the credit cycle.

The CreditPro database recognizes 12 industry classifications, which we assign to the general categories as follows. Utilities, Transportation and Telecoms are assigned to utilities, Finance Institutions, Insurance and Real Estate to financials and the rest to industrials (Energy & Natural Resources, Forest & Building Products / Homebuilders, Consumer Services, Leisure & Media, Healthcare / Chemicals, High Technology / Computers / Office Equipment, Aerospace / Automobiles / Capital Goods / Metals).

We organize the data in terms of independent durations. Each firm that for example began the sample holding a BB- rating got upgraded to B+ after 600 days, got downgraded to C after 2000 days and has kept this rating over the rest of the sample produces the exactly corresponding independent rating durations, taking special care to take into account the inherent censoring regarding the last one. In total, the data consists of 16,938 rating durations.

Also our sample is left censored. That is for the firms that held ratings on January 1st 1981 we also know that they had not already failed to an available destination. January 1st 1981 was not a date with an intense flurry of rating activity; it just happened to mark the start of the S&P database. Thus, we follow standard practice in evaluating the impact of left censoring, by omitting the first day of the data and concentrating on the rest. We thus discard 1360 observations the results are almost identical to the estimation without the truncation correction**.**

## Appendix B

### Credit Spreads and Default Probabilities

Harrison and Kreps (1979) prove that absence of arbitrage is equivalent to the existence of an equivalent martingale measure, which in turn implies the existence of a strictly positive discount factor, $M_{t+1}$ or pricing kernel such that for any traded asset the following relationship holds,

$$p_t = E_t(M_{t+1} X_{t+1})$$

$P_t$ is today's price and $X_t$ tomorrow's payoff. This relationship holds for the price of a one period risk free asset, $p^f$, that we call a government bond and for the price of a one period defaultable security, $p^r$, which we call a corporate bond. Both these assets have a unit next period payoff, the difference arising from the fact that the corporate security pays out only in the case of no default[68], i.e. >t+1. Thus,

$$p_t^f = E_t(M_{t+1})$$

$$p_t^r = E_t(M_{t+1} 1_{\{\tau > t+1\}})$$

The indicator function takes the value of one when default does not happen in period t+1. From these two equations we derive an expression for the yield spread between risky and risk-free bonds.

[68] We assume a zero recovery rate in the case of default.

$$(1+y_t^f)^{-1} = E_t(M_{t+1})$$
$$(1+y_t^r)^{-1} = E_t(M_{t+1}1_{\{\tau>t+1\}})$$

We rewrite the equation for the risk free asset as

$$(1+y_t^f)^{-1} = E_t(M_{t+1}1_{\{\tau>t+1\}}) + E_t(M_{t+1}1_{\{\tau=t+1\}})$$

Subtracting, we get

$$\frac{y_t^r - y_t^f}{(1+y_t^r)(1+y_t^r)} = E_t(M_{t+1}1_{\{\tau=t+1\}}) = E_t(M_{t+1})E_t(1_{\{\tau=t+1\}}) + \mathrm{cov}(M_{t+1}, 1_{\{\tau=t+1\}})$$

This formula is approximated for small y's as follows.[69]

$$y_t^r - y_t^f = E_t(M_{t+1}1_{\{\tau=t+1\}}) = \frac{1}{1+y_t^f}E_t(1_{\{\tau=t+1\}}) + \mathrm{cov}(M_{t+1}, 1_{\{\tau=t+1\}})$$

The credit yield spread can be interpreted as the risk neutral default probability, exploiting the martingale measure equivalence with the stochastic discount factor, or as the empirical default probability plus the covariance of the pricing kernel with the default event. Intuitively, using a marginal utility based SDF argument, if default happens when marginal utility is high, the spread is higher, and i.e. the risky bond is cheaper in order to compensate for this unfavorable property. In the defaultable bond modeling literature one either assumes independence under the risk *neutral* measure of the spot rate with the default process or handles dependence by adjusting the discount rate by the instantaneous default probability, deriving something close to (18).

## Appendix C

[69] An approximation that becomes exact in continuous time.

### Examination of Endogeneity/ Predictability of Exogenous Variables

We suggest a series of Granger causality tests, which test for precedence and information content. As a proxy for the evolution of the transition intensity matrix we will use the log ratio of upgrade to downgrade probabilities calculated nonparametrically. The following shows that this is the correct functional form to contemplate using previous formulas:

$$\log(\frac{p_i^u}{p_i^d}) = \log[\frac{\exp(\gamma_i^u X(t))\sum_{j=1}^{J_u}\exp(\eta_{ij})}{1+\exp(\gamma_i^u X(t))\sum_{j=1}^{J_u}\exp(\eta_{ij})+\exp(\gamma_i^d X(t))\sum_{j=1}^{J_d}\exp(\eta_{ij})} \times \{\frac{\exp(\gamma_i^d X(t))\sum_{j=1}^{J_d}\exp(\eta_{ij})}{1+\exp(\gamma_i^u X(t))\sum_{j=1}^{J_u}\exp(\eta_{ij})+\exp(\gamma_i^d X(t))\sum_{j=1}^{J_d}\exp(\eta_{ij})}\}^{-1}]$$

Simplifying we derive an expression that relates the log upgrade downgrade probability ratio with an affine function of the relevant state variables. Thus, we will perform Granger causality tests using the coefficients derived from regressions of the time series of nonparametric log ratios on the conditioning variables. The actual bivariate regressions that are run are

$$\log(\frac{\sum_{i=1}^{I_u} p_i^u(t)}{\sum_{i=1}^{I_d} p_i^d(t)}) = a_0 + a_1 \log(\frac{\sum_{i=1}^{I_u} p_i^u(t-1)}{\sum_{i=1}^{I_d} p_i^d(t-1)}) + .. + \beta_1 X(t-1) + .. + \varepsilon(t)$$

$$X(t) = a_0 + \beta_1 \log(\frac{\sum_{i=1}^{I_u} p_i^u(t-1)}{\sum_{i=1}^{I_d} p_i^d(t-1)}) + .. + a_1 X(t-1) + .. + \varepsilon(t)$$

We use three lags in the parameterization employed. The reported F-statistics are the Wald Statistics for the joint hypothesis that

$$\beta_1 = \beta_2 = \beta_3$$

Hence, the null hypothesis is that X *does not* Granger cause the proxy for the transition intensity process in the first regression and the other way around in the second.

# Appendix D

## Term Structure Stylized Facts

The most popular variable in term structure stylized fact studies is the slope, equivalently the term spread. Studies documenting the predictive ability of this variable regarding business cycle forecasting, for example Estrella and Miskin (1999), report that a low slope is the winner in a recession forecasting horse race. Moreover, as Seppala (1999) reports, the term spread is clearly countercyclical and as the term spread increases we are traversing from the peak to the trough of the business cycle.

Simple correlations for our sample period, between the spot rate and the slope on the one hand and aggregate or more specifically credit market indicators on the other hand, indicate that the spot rate is procyclical and the slope countercyclical. Similar results are obtained by restricting the sample from January 1985 till March 1995. This is the sample used by Duffee (1999). Nevertheless, correlations between the levels of the term structure variables and industrial production growth do not concentrate on the appropriate business cycle frequency. Thus in order to focus on the business cycle properties we follow the practice of the real business cycle literature and examine the correlation between Hodrick-Prescott filtered variables. We conclude that the spot rate is actually procyclical and the slope countercyclical for both Duffee's sample and ours[70]. Cross-correlations over time display phase shifts that accord with the established pattern and corroborate the results that will follow (Table IV.1.c.1). In essence, a high spot rate is a leading indicator of recession and a large slope is a leading indicator of expansion. In other words, when the interest rates are increasing we are close or at the peak of the cycle, with short rates increasing more than long rates, i.e. the slope going down.

[70] Duffee calculates correlations between log-differences that tend to emphasize overly high frequencies.

## Appendix E

## Matrices

### Matrix 1a: One-Year Constant Intensity Matrix, NR adjustment

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 94.46% | 4.77% | 0.60% | 0.10% | 0.07% | 0.01% | 0.00% | 0.00% |
| AA | 0.49% | 91.79% | 6.94% | 0.59% | 0.07% | 0.09% | 0.02% | 0.01% |
| A | 0.10% | 1.87% | 91.99% | 5.22% | 0.55% | 0.25% | 0.01% | 0.01% |
| BBB | 0.04% | 0.32% | 5.47% | 88.19% | 4.96% | 0.83% | 0.07% | 0.11% |
| BB | 0.04% | 0.14% | 0.84% | 6.75% | 83.88% | 7.13% | 0.72% | 0.50% |
| B | 0.01% | 0.07% | 0.27% | 0.60% | 5.62% | 84.79% | 4.81% | 3.84% |
| CCC | 0.12% | 0.01% | 0.51% | 0.46% | 1.70% | 8.91% | 54.01% | 34.28% |

### Matrix 1b: S&P One-Year Transition Matrix, NR adjustment

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 92.83% | 6.50% | 0.56% | 0.06% | 0.06% | 0.00% | 0.00% | 0.00% |
| AA | 0.63% | 91.87% | 6.64% | 0.65% | 0.06% | 0.11% | 0.04% | 0.00% |
| A | 0.08% | 2.26% | 91.65% | 5.11% | 0.61% | 0.23% | 0.01% | 0.04% |
| BBB | 0.05% | 0.27% | 5.84% | 87.76% | 4.74% | 0.98% | 0.16% | 0.22% |
| BB | 0.04% | 0.11% | 0.64% | 7.85% | 81.13% | 8.27% | 0.89% | 1.06% |
| B | 0.00% | 0.11% | 0.30% | 0.42% | 6.75% | 83.08% | 3.86% | 5.49% |
| CCC | 0.19% | 0.00% | 0.38% | 0.75% | 2.44% | 12.03% | 60.71% | 23.50% |

### Matrix 1c: Two-Year Constant Intensity Matrix

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 89.26% | 8.89% | 1.45% | 0.24% | 0.14% | 0.02% | 0.00% | 0.00% |
| AA | 0.92% | 84.41% | 12.79% | 1.43% | 0.20% | 0.20% | 0.03% | 0.02% |
| A | 0.20% | 3.46% | 85.05% | 9.45% | 1.23% | 0.53% | 0.04% | 0.04% |
| BBB | 0.07% | 0.69% | 9.92% | 78.40% | 8.62% | 1.82% | 0.18% | 0.30% |
| BB | 0.08% | 0.29% | 1.88% | 11.70% | 71.12% | 12.15% | 1.34% | 1.44% |
| B | 0.02% | 0.15% | 0.59% | 1.45% | 9.59% | 72.72% | 6.71% | 8.76% |
| CCC | 0.18% | 0.05% | 0.81% | 0.84% | 2.87% | 12.49% | 29.61% | 53.14% |

**Matrix 2a: One-Year Transition Matrix (Conditional on a 'bad' state)**

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 89.66% | 8.62% | 1.34% | 0.22% | 0.14% | 0.02% | 0.00% | 0.00% |
| AA | 0.52% | 85.33% | 12.37% | 1.35% | 0.18% | 0.19% | 0.04% | 0.02% |
| A | 0.11% | 1.99% | 86.48% | 9.58% | 1.23% | 0.54% | 0.04% | 0.03% |
| BBB | 0.04% | 0.34% | 5.80% | 82.34% | 9.12% | 1.88% | 0.20% | 0.28% |
| BB | 0.03% | 0.10% | 0.60% | 4.72% | 78.02% | 13.52% | 1.63% | 1.39% |
| B | 0.01% | 0.05% | 0.20% | 0.39% | 3.99% | 78.62% | 8.23% | 8.51% |
| CCC | 0.08% | 0.01% | 0.32% | 0.28% | 1.03% | 5.69% | 40.72% | 51.86% |

**Matrix 2b: One-Year Transition Matrix (Conditional on a 'good' state)**

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 95.81% | 3.65% | 0.41% | 0.07% | 0.05% | 0.00% | 0.00% | 0.00% |
| AA | 0.42% | 93.75% | 5.28% | 0.41% | 0.05% | 0.07% | 0.02% | 0.00% |
| A | 0.09% | 1.61% | 93.62% | 4.10% | 0.40% | 0.19% | 0.00% | 0.00% |
| BBB | 0.03% | 0.26% | 4.71% | 90.33% | 3.96% | 0.59% | 0.05% | 0.07% |
| BB | 0.04% | 0.14% | 0.82% | 7.09% | 86.92% | 4.36% | 0.42% | 0.21% |
| B | 0.00% | 0.08% | 0.27% | 0.62% | 6.02% | 88.00% | 3.15% | 1.86% |
| CCC | 0.15% | 0.01% | 0.60% | 0.53% | 2.02% | 10.71% | 67.01% | 18.97% |

## Appendix F

## Tables

### Table II.2.1: “BB barrier” cumulative probabilities

| | 1y | 2y | 3y | 4y | 5y |
|---|---|---|---|---|---|
| AAA | 0.07% | 0.15% | 0.24% | 0.33% | 0.42% |
| AA | 0.17% | 0.38% | 0.62% | 0.88% | 1.16% |
| A | 0.75% | 1.55% | 2.35% | 3.08% | 3.72% |
| BBB | 5.39% | 9.01% | 11.31% | 12.65% | 13.31% |

### Table III.b.1: Conditional 1-month lower/upper third Credit State vs. Kaplan-Meier Comparison

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | Ns | ns | ns | -0.01% | -0.01% | ns | ns | ns |
| AA | -0.02% | -0.19% | 0.19% | ns | 0.01% | ns | ns | ns |
| A | Ns | ns | -0.36% | 0.33% | ns | 0.02% | ns | ns |
| BBB | 0.01% | ns | ns | -0.23% | 0.23% | 0.04% | 0.01% | ns |
| BB | Ns | ns | ns | -0.10% | -0.35% | 0.36% | 0.07% | 0.03% |
| B | Ns | ns | -0.01% | ns | -0.14% | -0.53% | 0.44% | 0.22% |
| CCC | Ns | ns | ns | ns | ns | ns | ns | 0.98% |

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | 0.14% | -0.13% | ns | -0.01% | ns | ns | ns | ns |
| AA | Ns | 0.21% | -0.16% | -0.02% | ns | -0.01% | ns | ns |
| A | Ns | ns | 0.18% | -0.13% | -0.03% | -0.01% | ns | ns |
| BBB | Ns | ns | ns | 0.24% | -0.16% | -0.05% | ns | -0.01% |
| BB | Ns | ns | ns | ns | 0.25% | -0.20% | -0.03% | ns |
| B | Ns | -0.01% | ns | ns | ns | 0.23% | -0.19% | -0.11% |
| CCC | -0.01% | ns | ns | ns | ns | 0.80% | ns | ns |

### Table III.b.2: Conditional 1-month lower/upper third S&P Implied Volatility vs. Kaplan-Meier Comparison

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|
| AAA | ns | ns | ns | -0.01% | ns | ns | ns | ns |
| AA | -0.03% | 0.11% | ns | -0.03% | ns | -0.01% | ns | ns |
| A | ns | ns | 0.16% | -0.12% | -0.03% | -0.01% | ns | ns |
| BBB | ns | ns | ns | 0.18% | -0.13% | -0.03% | ns | -0.01% |
| BB | ns | ns | ns | ns | 0.14% | -0.15% | -0.02% | -0.02% |
| B | ns | -0.01% | ns | ns | 0.17% | 0.11% | -0.22% | -0.06% |
| CCC | -0.01% | ns | -0.06% | -0.04% | ns | 1.11% | ns | -1.10% |

| | AAA | AA | A | BBB | BB | B | CCC | D |
|---|---|---|---|---|---|---|---|---|

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| AAA | ns | ns | ns | ns | -0.01% | ns | ns | ns |
| AA | ns | -0.14% | ns | ns | ns | ns | ns | ns |
| A | ns | ns | -0.24% | 0.17% | 0.03% | ns | ns | ns |
| BBB | ns | ns | 0.06% | -0.29% | 0.18% | 0.05% | 0.01% | -0.01% |
| BB | ns | -0.01% | ns | ns | -0.16% | 0.15% | 0.06% | ns |
| B | ns | -0.01% | ns | ns | ns | ns | 0.11% | ns |
| CCC | -0.01% | ns | ns | ns | ns | -0.43% | ns | ns |

### Table IV.1.b.1. Granger Causality Tests

### Investment Grade Section

| Null Hypothesis: | F-Statistic | Probability |
|---|---|---|
| Garch does not Granger Cause Prob | 2.03 | 0.11 |
| Prob does not Granger Cause Garch | 1.36 | 0.26 |
| Prob does not Granger Cause Slope | 2.16 | 0.10 |
| Slope does not Granger Cause Prob | 0.75 | 0.53 |
| Vol does not Granger Cause Prob | 0.77 | 0.51 |
| Prob does not Granger Cause Vol | 0.54 | 0.66 |
| Spot does not Granger Cause Prob | 1.50 | 0.22 |
| Prob does not Granger Cause Spot | 0.31 | 0.82 |
| Prob does not Granger Cause CS | 6.54 | 0.00 |
| CS does not Granger Cause Prob | 3.45 | 0.02 |

### Speculative Grade Section

| Null Hypothesis: | F-Statistic | Probability |
|---|---|---|
| Prob does not Granger Cause CS | 6.87 | 0.00 |
| CS does not Granger Cause Prob | 1.53 | 0.21 |
| Garch does not Granger Cause Prob | 0.91 | 0.44 |
| Prob does not Granger Cause Garch | 0.64 | 0.59 |
| Prob does not Granger Cause Slope | 0.85 | 0.47 |
| Slope does not Granger Cause Prob | 1.30 | 0.28 |
| Vol does not Granger Cause Prob | 1.10 | 0.35 |
| Prob does not Granger Cause Vol | 1.51 | 0.22 |
| Spot does not Granger Cause Prob | 2.70 | 0.05 |
| Prob does not Granger Cause Spot | 0.62 | 0.60 |

### Table IV.1.b.2: Credit State Constituent Series and Baa Spread Analysis

### Ceteris paribus ± change of the state variables

| Credit State | | | | | |
|---|---|---|---|---|---|
| AA | 95.20% | 105.24% | 93.99% | 82.74% | 92.49% |
| A | 81.77% | 84.39% | 75.24% | 66.09% | 67.30% |
| BBB | 56.10% | 62.55% | 52.89% | 43.23% | 49.66% |
| BB | 62.89% | 64.01% | 54.25% | 44.49% | 45.34% |
| B | 71.97% | 67.29% | 58.36% | 49.44% | 43.35% |

| | | | | | |
|---|---|---|---|---|---|
| CCC | 82.66% | 86.52% | 73.35% | 60.18% | 61.38% |

| | Baa Spread | | | | |
|---|---|---|---|---|---|
| BBB | 42.36% | 43.23% | 52.89% | 62.55% | 63.17% |

**Log Odds Derivatives along the Diagonal (*'s denote significance)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| State | 0.1169 | 0.275* | 0.089 | 0.2459* | 0.4173* | 0.4235* |
| Baa Spread | 0.1059 | 0.4989* | -0.8619* | -0.2499 | -0.0316 | 0.4098 |

**Ceteris paribus ± change of the state variables**

| | nWAR | | | | |
|---|---|---|---|---|---|
| AA | 0.9359 | 0.8906 | 0.9402 | 0.9898 | 0.9442 |
| A | 0.7621 | 0.8145 | 0.7544 | 0.6943 | 0.7464 |
| BBB | 0.5557 | 0.5834 | 0.519 | 0.4547 | 0.4822 |
| BB | 0.575 | 0.5996 | 0.5363 | 0.473 | 0.4971 |
| B | 0.6118 | 0.6373 | 0.5792 | 0.5211 | 0.546 |
| CCC | 0.7913 | 0.8251 | 0.7387 | 0.6523 | 0.6782 |

| | IP growth | | | | |
|---|---|---|---|---|---|
| AA | 0.9535 | 0.8906 | 0.9402 | 0.9898 | 0.9234 |
| A | 0.7918 | 0.8145 | 0.7544 | 0.6943 | 0.7127 |
| BBB | 0.5845 | 0.5834 | 0.519 | 0.4547 | 0.4529 |
| BB | 0.55 | 0.5996 | 0.5363 | 0.473 | 0.5225 |
| B | 0.603 | 0.6373 | 0.5792 | 0.5211 | 0.555 |
| CCC | 0.7572 | 0.8251 | 0.7387 | 0.6523 | 0.7193 |

**Log Odds Derivatives along the Diagonal (*'s denote significance)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Baa Spread | -0.014 | 0.629* | -0.647* | -0.424 | -0.339 | 0.240 |
| nWAR | -0.043 | 0.041 | 0.0502* | 0.042 | 0.0914* | 0.1339* |
| IP | 0.153 | 0.1041* | -0.003 | 0.075 | 0.091* | -0.002 |

**Table IV.1.b.3: Level and Slope of the Term Structure Analysis**

**Log Odds Derivatives along the Diagonal (*'s denote significance)**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | 0.5959 | -0.1786 | -0.0124 | -0.0982* | -0.1498* | -0.0843* | -0.1594 |
| Slope | -0.0588 | -0.3914 | -0.1678* | -0.0836 | -0.1625 | -0.0182 | 0.3075 |

**Table IV.1.b.4: Level and Slope of the Term Structure Analysis: Restricted Samples January 1985-March 1995, January 1983-December 1998**

**Log Odds Derivatives along the Diagonal (*'s denote significance)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Spot | 0.4817 | -0.1096 | -0.0722 | -0.4824* | -0.5241* | -0.7075* |
| Slope | 0.614 | -0.0341 | 0.0022 | -0.5896* | -0.5048* | -0.56 |

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Spot | -0.1892 | -0.1919* | -0.1060 | -0.0529* | -0.2775* | -0.2952* |
| Slope | -0.3598 | -0.3261* | -0.1354 | -0.1088 | -0.0765 | 0.0932 |

**Table IV.1.b.5: Level and Curvature of the Term Structure Analysis**

**Log odds derivatives along the diagonal (*'s denote significance)**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | 0.7542 | -0.0807 | 0.0139 | -0.0921* | -0.161* | -0.2861* | -0.3047* |
| GARCH | -1.6099 | 0.0383 | -0.0775 | -0.0257 | 0.4308 | 1.2566* | 0.963 |

**Log odds derivatives (*'s denote significance), Sensitivity Restrictions**

| | Spot | GARCH |
|---|---|---|
| High Grade | -0.083* | 0.4604 |
| Low Grade | -0.3769* | 2.9542* |

**Table IV.1.b.6: Real Rate and Nominal Slope of the Term Structure Analysis**

**Sensitivity Parameters**

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | 0.0774 | 0.0022 | 0.0511 | 0.0386 | 0.1377 | 0.0373 |
| | 0.0266 | 0.0484 | 0.0213 | 0.0383 | 0.0282 | 0.0547 |
| Low Grade | -0.0894 | -0.0731 | -0.1639 | 0.145 | -0.0373 | -0.0497 |
| | 0.0358 | 0.0538 | 0.0365 | 0.0529 | 0.0204 | 0.0321 |

**Semielasticities of Conditional Downgrade Probabilities**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| real rate | 8.55% | 8.35% | 7.36% | 3.11% | 2.59% | 4.88% | 2.81% |
| | 3.43% | 3.33% | 2.96% | 1.82% | 1.86% | 1.24% | 0.71% |

### Log odds derivatives along the diagonal (*'s denote significance)

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Real | -0.0849 | -0.0396 | -0.0274 | -0.1136* | -0.1335* | 0.079 |
| Slope | -0.2367 | -0.0964 | 0.0132 | -0.0434 | 0.0968 | 0.5238* |

### Table IV.1.b.7: Spot Rate and S&P 500 Return Analysis

### Sensitivity Parameters

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | 0.0537 | 0.0463 | 0.1274 | 0.0108 | 0.2093 | -0.0039 |
| | 0.0297 | 0.0120 | 0.0223 | 0.0089 | 0.0322 | 0.0119 |
| Low Grade | -0.0263 | -0.0032 | -0.2096 | -0.0030 | 0.1136 | 0.0104 |
| | 0.0328 | 0.0112 | 0.0342 | 0.0110 | 0.0188 | 0.0068 |

### Log odds derivatives (*'s denote significance)

| | High Grade | Low Grade |
|---|---|---|
| Spot | -0.0737 | 0.0355* |
| S&P 500 | -0.3232* | -0.0134 |

### Table IV.1.b.8: Spot Rate and S&P 500 Implied Volatility Analysis

### Sensitivity Parameters

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | 0.054 | 0.0257 | 0.0675 | 0.067 | 0.1442 | 0.1039 |
| | 0.0221 | 0.022 | 0.0177 | 0.0155 | 0.0221 | 0.0205 |
| Low Grade | -0.0323 | -0.0093 | -0.166 | -0.0348 | 0.0303 | 0.0497 |
| | 0.0262 | 0.0253 | 0.0293 | 0.0256 | 0.0143 | 0.0126 |

### Ceteris paribus ± change of the state variables

| | Vol | | | | |
|---|---|---|---|---|---|
| AAA | 1.26% | -0.11% | 1.35% | 2.80% | 1.44% |
| AA | 1.75% | 1.03% | 1.88% | 2.73% | 2.02% |
| A | 7.65% | 6.46% | 8.31% | 10.16% | 9.01% |
| BBB | 48.34% | 45.81% | 51.97% | 58.13% | 55.58% |
| BB | 50.65% | 47.55% | 53.38% | 59.22% | 56.10% |
| B | 57.32% | 55.45% | 60.99% | 66.53% | 64.52% |
| CCC | 73.94% | 72.23% | 76.77% | 81.32% | 79.38% |

### Log odds derivatives along the diagonal (*'s denote significance)

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | 0.452 | -0.0746 | -0.0047 | -0.0734* | -0.102* | -0.1636* | -0.2336* |
| Vol | 1.4056 | -0.0149 | 0.1 | -0.1001* | -0.089* | -0.0585 | -0.0889 |

**Log odds derivatives (*'s denote significance), Sensitivity Restrictions**

| | High Grade | Low Grade |
|---|---|---|
| Spot | -0.048 | -0.0258 |
| Vol | -0.2789* | -0.0644* |

**Table IV.1.b.9: Level, S&P 500, HML Return Analysis**

**Log odds derivatives along the diagonal (*'s denote significance)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Spot | 0.0923 | -0.0607 | -0.1262* | -0.181* | -0.2789* | -0.3527* |
| S&P 500 | 0.0039 | 0.0413 | 0.04485* | -0.0127 | -0.0195 | -0.0016 |
| HML | 0.1685 | 0.1624* | -0.0184 | -0.0155 | 0.1066* | 0.0442 |

**Log odds derivatives (*'s denote significance), Sensitivity Restrictions**

| | Spot | S&P 500 | HML |
|---|---|---|---|
| High Grade | -0.06074* | 0.04129* | 0.16244* |
| Low Grade | -0.1737* | 0.0024 | -0.0074 |

**Table IV.2.b.1: In-sample Measures of Fit**

**1991-1998**

| 1y | Overall | Profit | Inv Grade | Spec Grade |
|---|---|---|---|---|
| KaplanMeier | 0.6831 | 0.4969 | 0.2344 | 0.4486 |
| Martingale | 0.7425 | 0.6465 | 0.2287 | 0.5138 |
| Exponential | 0.687 | 0.5015 | 0.2398 | 0.4472 |
| StateSpread | 0.5808 | 0.4819 | 0.1638 | 0.417 |
| SpotSlope | 0.6193 | 0.4789 | 0.1855 | 0.4338 |
| SpotSPVol | 0.609 | 0.4808 | 0.1867 | 0.4223 |
| SpotIRGarch | 0.6054 | 0.4534 | 0.1937 | 0.4116 |
| SpotSP500 | 0.6283 | 0.4819 | 0.2013 | 0.4269 |
| SpotSlopeVol | 0.6042 | 0.4797 | 0.174 | 0.4302 |
| SpotSlopeGarch | 0.5923 | 0.4551 | 0.1728 | 0.4195 |
| SpotSlopeHML | 0.6113 | 0.4715 | 0.1825 | 0.4288 |
| SpotSPHML | 0.6228 | 0.4729 | 0.1999 | 0.4229 |
| SpotGarchVol | 0.5964 | 0.4618 | 0.184 | 0.4124 |
| Cscomponents | 0.5893 | 0.4765 | 0.1747 | 0.4146 |

**1991-1993**

| 1y | Overall | Profit | Inv Grade | Spec Grade |
|---|---|---|---|---|
| KaplanMeier | 0.791 | 0.6434 | 0.2314 | 0.5596 |
| Martingale | 0.8496 | 0.7278 | 0.2795 | 0.5701 |
| Exponential | 0.7832 | 0.6411 | 0.2314 | 0.5518 |
| StateSpread | 0.5775 | 0.3224 | 0.1947 | 0.3828 |

| | | | | |
|---|---|---|---|---|
| SpotSlope | 0.7356 | 0.6132 | 0.2107 | 0.5249 |
| SpotSPVol | 0.7184 | 0.5995 | 0.2064 | 0.512 |
| SpotIRGarch | 0.6904 | 0.5672 | 0.2021 | 0.4883 |
| SpotSP500 | 0.7318 | 0.6142 | 0.211 | 0.5208 |
| SpotSlopeVol | 0.7231 | 0.6017 | 0.2046 | 0.5185 |
| SpotSlopeGarch | 0.6861 | 0.5561 | 0.1973 | 0.4888 |
| SpotSlopeHML | 0.7162 | 0.5908 | 0.2057 | 0.5105 |
| SpotSPHML | 0.7151 | 0.5947 | 0.2073 | 0.5078 |
| SpotGarchVol | 0.6891 | 0.567 | 0.2007 | 0.4884 |
| Cscomponents | 0.5852 | 0.3333 | 0.2058 | 0.3794 |

**1994-1998**

| 1y | Overall | Profit | Inv Grade | Spec Grade |
|---|---|---|---|---|
| KaplanMeier | 0.6172 | 0.4076 | 0.2363 | 0.3809 |
| Martingale | 0.6771 | 0.5969 | 0.1977 | 0.4794 |
| Exponential | 0.6282 | 0.4163 | 0.2449 | 0.3833 |
| StateSpread | 0.5829 | 0.5793 | 0.145 | 0.4379 |
| SpotSlope | 0.5483 | 0.3969 | 0.17 | 0.3783 |
| SpotSPVol | 0.5423 | 0.4085 | 0.1748 | 0.3675 |
| SpotIRGarch | 0.5535 | 0.384 | 0.1886 | 0.3649 |
| SpotSP500 | 0.5651 | 0.4011 | 0.1955 | 0.3696 |
| SpotSlopeVol | 0.5317 | 0.4053 | 0.1554 | 0.3763 |
| SpotSlopeGarch | 0.535 | 0.3934 | 0.1578 | 0.3772 |
| SpotSlopeHML | 0.5472 | 0.3988 | 0.1683 | 0.3789 |
| SpotSPHML | 0.5665 | 0.3986 | 0.1954 | 0.3711 |
| SpotGarchVol | 0.5399 | 0.3976 | 0.1738 | 0.3661 |
| Cscomponents | 0.5919 | 0.5639 | 0.1557 | 0.4361 |

**Table IV.2.b.2: Out-of-sample Measures of Fit**

**1997-1998**

| 1m | Overall | Profit | Inv Grade | Spec Grade |
|---|---|---|---|---|
| KaplanMeier | 0.1873 | 1.7751 | 0.0449 | 0.1424 |
| Martingale | 0.1982 | 2.1853 | 0.0366 | 0.1616 |
| Exponential | 0.1803 | 1.7216 | 0.0433 | 0.1369 |
| SpotSlopeVol | 0.1783 | 1.724 | 0.038 | 0.1403 |
| SpotGarchVol | 0.1783 | 1.7277 | 0.0402 | 0.1381 |
| CsComponents | 0.1784 | 1.8484 | 0.0346 | 0.1438 |

**Table V.1.b.i.1: Time Acceleration Shape Parameters**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Upgrade Acceleration | 0 | 1.0744 | 1.1321 | 1.361 | 1.568 | 1.6689 | 1.2998 |
| | 0 | -0.1866 | -0.0705 | -0.0518 | -0.0584 | -0.0594 | -0.0912 |
| Downgrade Acceleration | 1.176 | 1.2218 | 1.1199 | 0.9871 | 1.1915 | 1.105 | 0.615 |
| | -0.1148 | -0.0525 | -0.0401 | -0.0406 | -0.0461 | -0.0387 | -0.0309 |

**Table V.1.b.ii.1: Level and Slope Analysis under Time Dependence**

**Sensitivity Parameters**

| | | | | | alpha | beta |
|---|---|---|---|---|---|---|
| High Grade | 0.1373 | 0.1107 | 0.1879 | 0.2082 | 1.3512 | 1.2171 |
| | 0.0259 | 0.0533 | 0.0449 | 0.0163 | 0.035 | 0.0281 |
| Low Grade | -0.085 | -0.0201 | 0.0378 | 0.0053 | 1.5587 | 0.992 |
| | 0.023 | 0.0425 | 0.038 | 0.0161 | 0.0366 | 0.023 |

**Semielasticities of Conditional Downgrade Probabilities (1-month)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Level | 1.07% | 4.11% | 0.77% | 4.00% | 2.64% | 1.06% |
| | 5.54% | 2.39% | 1.34% | 1.32% | 0.60% | 0.86% |
| Slope | 2.67% | 9.48% | 0.61% | 4.09% | 1.11% | -1.39% |
| | 6.66% | 4.53% | 1.73% | 2.32% | 1.24% | 1.75% |

**Semielasticities of Conditional Downgrade Probabilities (12-months)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Level | 0.68% | 3.70% | 1.42% | 6.60% | 6.61% | 3.69% |
| | 4.42% | 2.24% | 2.47% | 2.53% | 1.94% | 2.82% |
| Slope | 1.70% | 8.54% | 1.12% | 6.74% | 2.77% | -4.86% |
| | 4.98% | 4.40% | 3.16% | 3.76% | 3.00% | 5.56% |

**Table V.2.1: Rating Momentum Sensitivity**

| Upward | Downward |
|---|---|
| 2.35 | 3.4542 |

**Table V.2.2: Level of the Term Structure Analysis under Rating Momentum**

| | | | Momentum | |
|---|---|---|---|---|
| High Grade | 0.1069 | 0.1174 | 2.6129 | 1.8357 |
| | 0.0221 | 0.0925 | 0.0134 | 0.1077 |
| Low Grade | -0.1016 | 0.0641 | 1.4342 | 1.7443 |
| | 0.0193 | 0.2413 | 0.0139 | 0.0674 |

**Semielasticities of Conditional Downgrade Probabilities**

| | BB | B | CCC |
|---|---|---|---|
| Momentum | 5.71% | 4.85% | 3.93% |
| | 0.32% | 0.27% | 0.29% |
| No Momentum | 9.64% | 8.82% | 7.80% |

| | | | |
|---|---|---|---|
| | 0.98% | 0.86% | 0.83% |
| Level | 7.33% | 6.70% | 4.21% |
| | 0.92% | 0.81% | 0.55% |

**Table V.3.1: Log Odds Derivatives under various Time Dependence Parameterizations (*'s denote significance)**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| **Exponential** | | | | | | |
| spot | 0.1226419 | -0.0386977 | -0.1000894* | -0.1105451* | -0.2159748* | -0.1889955 |
| slope | 0.0870545 | -0.152281 | -0.0900317 | -0.1386071 | -0.066006 | -0.024914 |
| garch | 0.1322897* | 0.0186233 | -0.0977293* | -0.0848953* | -0.0021135 | -0.0648686 |
| vol | 0.204576 | 0.363187 | -0.1596957 | 0.2352992 | 1.2274522* | 0.1020478 |
| **Weibull** | | | | | | |
| | AA | A | BBB | BB | B | CCC |
| spot | 0.0679006 | -0.0594449 | -0.0570806 | -0.1027648* | -0.2157075* | -0.1659523 |
| slope | 0.0129084 | -0.1750868 | -0.0454147 | -0.1230997 | -0.1090204 | -0.0327456 |
| garch | 0.1270759* | 0.016358 | -0.2361709* | -0.0858893* | 0.0069201 | -0.046687 |
| vol | 0.1209509 | 0.3565345 | -0.0658651 | 0.3842118 | 1.4087332* | 0.1089058 |
| **Gompertz** | | | | | | |
| | AA | A | BBB | BB | B | CCC |
| spot | 0.0768077 | -0.0790246 | -0.0506779 | -0.0961148 | -0.205535* | -0.1493134 |
| slope | 0.0197051 | -0.2071336 | -0.0283442 | -0.1170796 | -0.0681078 | -0.0111031 |
| garch | 0.1254234* | 0.0143791 | -0.0897934* | -0.0832361* | 0.0043617 | -0.0490702 |
| vol | 0.2195125 | 0.3821373 | -0.1554315 | 0.2953697 | 1.2760918* | 0.0961697 |
| **Cox** | | | | | | |
| | AA | A | BBB | BB | B | CCC |
| spot | 0.0643126 | -0.0792004 | -0.0523958 | -0.1001053 | -0.2411879* | -0.2096425* |
| slope | 0.0246429 | -0.20889656 | -0.0467866 | -0.1151276 | -0.1309161 | -0.1321904 |
| garch | 0.147174* | 0.0171679 | -0.0917769* | -0.0827503* | 0.003259 | -0.0549509 |
| vol | 0.1803707 | 0.5358581 | -0.0439971 | 0.3824276 | 1.5014789* | 0.2474768 |

**Table V.3.2: Rating Momentum Significance**

| | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|
| Upward | ns | -0.967 | -0.357 | ns | ns | ns |
| Downward | ns | ns | 0.222 | 0.789 | 0.884 | 1.106 |

**Table V.3.3: Akaike Information Criterion**

| | **Upgrade** | | | | **Downgrade** | | | |
|---|---|---|---|---|---|---|---|---|
| | **Exponential** | **Weibull** | **Loglog** | **Gompertz** | **Exponential** | **Weibull** | **Loglog** | **Gompertz** |
| **AA** | | | | | | | | |
| LogL | -108.28377 | -108.004 | -107.964 | -108.261 | -800.75315 | -779.17 | -775.166 | -793.19 |
| Ancillary | | 1.148841 | 0.853607 | 0.001173 | | 1.372661 | 0.603135 | 0.005362 |
| **AIC** | 218.56754 | 220.0079 | 219.9279 | 220.5222 | 1603.5063 | 1562.339 | 1554.332 | 1590.38 |
| **A** | | | | | | | | |
| LogL | -580.70889 | -576.745 | -574.783 | -580.705 | -1310.899 | -1294.25 | -1287.24 | -1308.98 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Ancillary | | 1.211045 | 0.767689 | -0.00019 | | 1.256398 | 0.685298 | 0.002448 |
| **AIC** | 1163.41778 | 1157.49 | 1153.565 | 1165.41 | 2623.798 | 2592.493 | 2578.485 | 2621.959 |
| **BBB** | | | | | | | | |
| LogL | -953.80677 | -922.781 | -913.166 | -945.468 | -1060.322 | -1056.23 | -1045.58 | -1059.45 |
| Ancillary | | 1.40843 | 0.597312 | 0.005775 | | 1.128899 | 0.752635 | -0.00213 |
| **AIC** | 1909.61354 | 1849.563 | 1830.332 | 1894.937 | 2122.644 | 2116.459 | 2095.153 | 2122.896 |
| **BB** | | | | | | | | |
| LogL | -923.67976 | -862.818 | 857.3583 | -890.538 | -1033.5836 | -1009.69 | -993.228 | -1026.45 |
| Ancillary | | 1.591964 | 0.516351 | 0.013973 | | 1.322266 | 0.609861 | 0.007127 |
| **AIC** | 1849.35952 | 1729.636 | -1710.72 | 1785.075 | 2069.1672 | 2023.378 | 1990.456 | 2056.893 |
| **B** | | | | | | | | |
| LogL | -959.33807 | -880.474 | -865.068 | -925.626 | -1301.3088 | -1277.39 | -1255.35 | -1293.72 |
| Ancillary | | -1.67599 | 0.485056 | 0.013855 | | 1.282271 | 0.636642 | 0.006739 |
| **AIC** | 1920.67614 | 1764.948 | 1734.136 | 1855.252 | 2604.6176 | 2558.771 | 2514.706 | 2591.438 |
| **CCC** | | | | | | | | |
| LogL | -200.72924 | -196.28 | -190.434 | -200.439 | -526.70081 | -523.088 | -487.458 | -508.984 |
| Ancillary | | 1.276077 | 0.57989 | 0.00396 | | 0.883729 | 0.703915 | -0.02578 |
| **AIC** | 403.45848 | 396.5598 | 384.8685 | 404.8777 | 1055.40162 | 1050.176 | 978.916 | 1021.968 |

**Table VI.1.1: Log Odds Derivatives, Utilities (*'s denote significance)**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | 0.27126 | 0.08403 | 0.16844 | 0.05515 | 0.09024 | -0.1398 | -1.6110* |
| Slope | -0.2941 | 0.41753 | -0.0324 | -0.3889* | -0.1471 | 0.09344 | -1.0635 |
| Vol | -0.4946 | 0.20356 | -0.1003 | -0.0891 | -0.0859 | 0.06877 | -0.5613 |

**Table VI.1.2: Log Odds Derivatives, Financials (*'s denote significance)**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | 0.15628 | -0.0068 | -0.3751 | -0.3542* | -0.5117* | -0.1954 | -0.6931 |
| Slope | 0.38269* | -0.2823 | -0.3681 | -0.1699 | -0.4470* | -0.0323 | -0.1734 |
| Vol | 0.02494 | -0.0887 | 0.01589 | -0.1118* | -0.2230* | -0.1332 | 0.10308 |

**Table VI.1.3: Log Odds Derivatives, Industrials (*'s denote significance)**

| | AAA | AA | A | BBB | BB | B | CCC |
|---|---|---|---|---|---|---|---|
| Spot | -0.1017 | -0.1492 | 0.0718 | -0.0365 | -0.1280* | -0.1640* | 0.08472 |
| Slope | -0.9942 | -0.4699 | -0.1775 | 0.11876 | -0.1316 | -0.027 | 0.51473* |
| Vol | 3.35158 | 0.14557 | 0.10073 | -0.0866* | -0.0176 | -0.0604 | -0.1201 |

**Table VI.1.4: Log Odds Derivatives, Cox Parameterization (*'s denote significance)**

| | Utilities | | | | | |
|---|---|---|---|---|---|---|
| | AA | A | BBB | BB | B | CCC |
| spot | 0.062 | 0.172 | 0.102 | 0.068 | 0.043 | -0.290 |
| slope | 0.391 | -0.067 | -0.346 | -0.108 | -0.086 | -0.600 |

| **Financials** | | | | | | |
|---|---|---|---|---|---|---|
| | **AA** | **A** | **BBB** | **BB** | **B** | **CCC** |
| spot | 0.244 | -0.216* | -0.233* | -0.436* | -0.086 | -0.393 |
| slope | 0.069 | -0.239 | -0.147 | -0.596* | 0.179 | -0.358 |

| **Industrials** | | | | | | |
|---|---|---|---|---|---|---|
| | **AA** | **A** | **BBB** | **BB** | **B** | **CCC** |
| spot | -0.168 | 0.155 | -0.034 | -0.046 | -0.114* | -0.180 |
| slope | -0.528 | 0.011 | 0.185 | -0.041 | -0.095 | -0.145 |

**Table VI.1.5: Log Odds Derivatives, Stratified Partial Likelihood (*'s denote significance)**

| | **AA** | **A** | **BBB** | **BB** | **B** | **CCC** |
|---|---|---|---|---|---|---|
| spot | 0.0093 | -0.0105855 | -0.03585 | -0.0936569 | -0.2209396* | -0.210198* |
| slope | -0.0697485 | -0.1170086 | -0.0427387 | -0.1424495 | -0.1173259 | -0.1777383 |
| garch | 0.093992 | 0.0300335 | -0.0945951* | -0.0928003* | 0.0136931 | -0.0760857 |
| vol | 0.2926288 | 0.4409856 | -0.1284104 | 0.2944984 | 1.4276294* | 0.2228225 |

**Table VI.1.1: Log Odds Derivatives with respect to the Level and the Slope of the Term Structure**

**Full Information**

| | | | | |
|---|---|---|---|---|
| High Grade | -0.057733124* | -0.07578628 | -0.173072665* | -0.15100776* |
| Low Grade | -0.171375976* | 0.013362037 | -0.08617739* | -0.146144206* |

**Restricted Information**

| | | | | |
|---|---|---|---|---|
| High Grade | -0.0395 | -0.0872 | -0.1216* | -0.1472 |
| Low Grade | -0.201* | 0.0451 | -0.0958* | -0.106 |

**Equality Tests: t-statistics**

| | | | | |
|---|---|---|---|---|
| High Grade | -0.5009 | 0.1428 | -1.1807 | -0.0367 |
| Low Grade | 0.6636 | -0.3694 | 0.2086 | -0.4129 |

**Table VI.1.2: Log Odds Derivatives with respect to the Level and the Curvature of the Term Structure and the Implied Volatility of the S&P Return**

**Full Information**

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | -0.064507942* | 0.42244773* | -0.0453386287* | -0.189406114* | 0.39632662 | -0.0882021067* |
| Low Grade | -0.085198064* | 0.370064472 | -0.014715831 | -0.211654763* | 0.47560264* | -0.026820788 |

**Restricted Information**

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | -0.0142 | 0.0484 | -0.0417 | -0.0956* | -0.0462 | -0.0796* |
| Low Grade | -0.1209* | 0.6397* | -0.0444 | -0.2755* | 0.9244* | -0.0631* |

**Equality Tests: t-statistics**

| | | | | | | |
|---|---|---|---|---|---|---|
| High Grade | -0.982 | 1.118 | -0.089 | -1.845 | 1.305 | -0.222 |
| Low Grade | 1.032 | -1.227 | 1.128 | 2.386 | -2.444 | 1.904 |

**Table VI.2.1: Unobservable Heterogeneity, Parameter Estimates**

| | AA | | A | | BBB | | BB | | B | | CCC | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Alpha | 1.903 | 2.760 | 2.030 | 2.699 | 1.358 | 1.490 | 1.921 | 5.590 | 1.782 | 1.876 | 1.374 | 5.273 |
| Intercept | 0.395 | 0.016 | 0.517 | 0.013 | 0.396 | 0.008 | 0.368 | 0.009 | 0.309 | 0.009 | -0.407 | 0.032 |
| Spot | 0.520 | 0.003 | 0.522 | 0.002 | 0.526 | 0.001 | 0.531 | 0.002 | 0.536 | 0.001 | 0.508 | 0.006 |

**Table VI.2.1: Unobservable Heterogeneity, Heterogeneity Distribution**

| | | | |
|---|---|---|---|
| support | 0.000 | 0.891 | 1.000 |
| | | 5.063 | |
| mass | 0.986 | 0.002 | 0.012 |
| | 0.009 | 0.139 | 0.143 |

**Table D.1: Cyclical behavior of Term Structure Variables: Cross Correlations with Industrial Production: Deviations from Trend**

| lag | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| spot | 0.66 | 0.62 | 0.51 | 0.37 | 0.25 | 0.14 | 0.06 | -0.03 | -0.12 | -0.16 | -0.22 | -0.29 | -0.33 |
| slope | -0.28 | -0.21 | -0.12 | -0.05 | -0.01 | 0.01 | 0.04 | 0.08 | 0.08 | 0.11 | 0.14 | 0.16 | 0.18 |
| spot | 0.59 | 0.51 | 0.41 | 0.29 | 0.17 | 0.06 | -0.02 | -0.10 | -0.15 | -0.19 | -0.22 | -0.26 | -0.29 |
| slope | -0.06 | 0.02 | 0.08 | 0.13 | 0.16 | 0.16 | 0.17 | 0.18 | 0.17 | 0.15 | 0.13 | 0.12 | 0.11 |

# Appendix G

# Figures

**Figure III.1.a.1: Credit State**

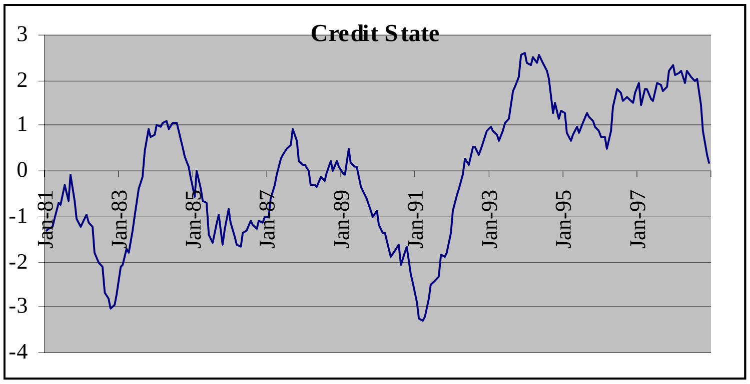
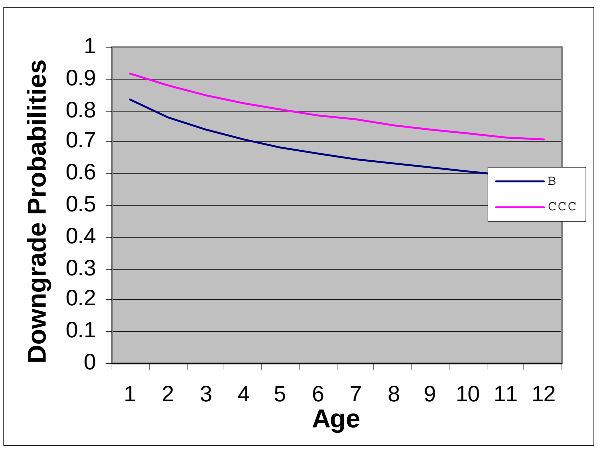
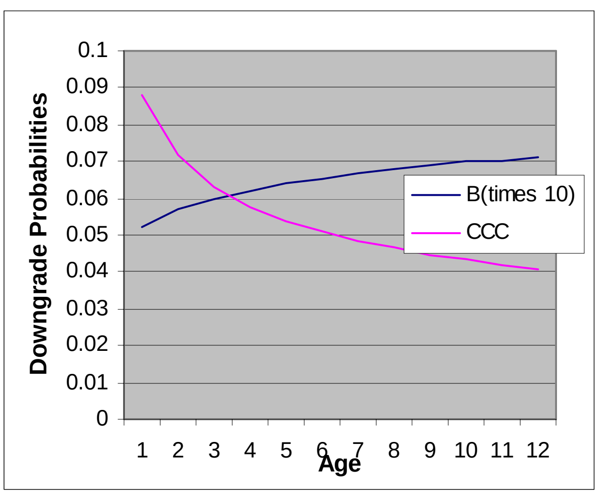



**Figure IV.1.b.ii.1,2: Conditional Downgrade Probabilities, Downgrade Probabilities**

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